Jaimes, Rocío

Explain discrete choice methods by animation videos

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Director: Bernhardt, José Alejandro

UNIVERSIDAD CATOLICA DE CORDOBA

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EXPLAIN DISCRETE CHOICE METHODS BY ANIMATION VIDEOS

AUTOR: ROCIO JAIMES
DIRECTOR: JOSÉ ALEJANDRO BERNHARDT MONTENEGRO

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Abstract

2020 was the year in which distance learning (also known as Virtual Education) gained importance, providing an analysis framework on existing tools in order to enhance their results. Our goal was to provide a tool that would assist students in understanding the Discrete Choice Methods, and for this we have started from an analysis of the existing types of education: Traditional Education, Virtual Education and Hybrid Education, summarizing the importance of the latter, and the positive results what can be obtained in relation to the other two. Educational videos, their elements and classifications were also analyzed, making a distinct emphasis on how to achieve Student Engagement through Working Memory, which can be stimulated through audiovisual channels. Then the types of Animated videos were discussed, of which one called Whiteboard Video was selected for our project. After that, a theoretical framework of the Discrete Choice Models was developed, its elements and the Utility Function that was used as a basis for the analysis, getting to explain the Multinomial Logit Model and the Blue Bus/Red Bus paradox. Finally, our experience preparing the video was shared, exposing both advantages and disadvantages of each alternative analized about wich animation program should be used and stating the decision made. The creative process was also shared along with the difficulties faced to achieve our outcome. The result was an Animated Whiteboard Video, which meets cognitive requirements according to what was analyzed. How this video influences the students, and if it can improve understanding remains to be tested in the future.

Keywords

Discrete Choice Methods, Discrete Choice Model, Multinomial Logit Model, Blue Bus/Red Bus paradox, Educational Video, types of Educational Videos, Elements Educational Videos, Virtual Education, Traditional Education, Hybrid Education, Animated Video, Witheboard Video, Student Engagement, Cognitive Load

Contents

List of acronyms

DCM: Discrete Choice Model

DM: Decision Maker

ED: Equivalent Differences

HE: Hybrid Education

IIA: Independece of Irrelevant Alternatives

MNL: Multinomial Logit Model

VE: Virual Education

TE: Traditional Education

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I. INTRODUCTION

The year 2020 will be historically remembered due to the extraordinary events that humanity had to live through, from the emergence of the COVID-19 outbreak, to the subsequent global need to take measures to prevent its spread, which ended in global isolation. Faced with such unexpectbed circumstances, schools and universities around the world were forced to reorganize their structures to continue with the planned educational program, trying that the year were not a complete loss for their students. It is within this framework that distance learning has acquired a new and more important presence in educational circles, seeking to avoid contact, prohibited due to the pandemic.

But this type of tutoring faces numerous limitations, which are increasingly recognized among educators. Instructors of all kinds were faced with the same situation: from the search for a platform that let them gives tutorials online, to the need to learn quickly how to handle it in order to resume classes as soon as possible, all this while they had to seek, in addition, how to make themselves understood by the students. It was seeing this whole situation that has led to think of different ways to help educators and contribute to give a dynamic boost to the education, assisting in the capture of the interest and improve the understanding of students.

Among the possible tools available, and based on personal experiences, we concluded that the videos were the ones that mostly helped the students to undertand or to embark themself in a new topic. If we considered that it worked well for us, we thought that maybe this way it could also work for others.

As a topic for our experimental video, we chose to expose and explain the Multinomial Logit Model, along with its properties, but before talking about the video, the development process and the tools used to create it, we will continue exposing the theoretical framework to help the reader's understanding.

1.1. Previous Studies

The idea of using videos as a supplement for lessons is not something new and has been analyzed and measured by numerous studies in the past. Mayer and Gallini (1990) carried out an investigation on the effect of images on students, exposing them to scientific works, which presents variations in its content: without illustrations, with static illustrations in certain parts, and with dynamic illustrations. Their study showed that visual content improved students' performance and understanding.

Since then it has been tested numerous times in different groups, for example on Business Statistics students (Haughton & Kelly, 2014), in students of the Operations classes (Prashar, 2015), also in Politics and International Relations students (Holland, 2013), Accounting (Martin, Evans, & Foster, 1995), among others. In all of them, improvements of different scales were observed, proving that the use of videos as an educational tool was positive.

Between the journals a recurring topic was the importance of the purpose for which the video will be used, such as:

- Used after a class to reinforce knowledge on key points.
- Applied to teach a new theme from scratch.
- Used to give a general review of a certain topic.

These are mere examples, used to clarify the importance in defined it accurately since it will be the starting point that will guide the content and the form that the video will take, as well as the model that it would be advisable to implement. An unclear objective can end in a vain effort and an indifferent reaction from the students.

1.2. Educational video - Summary of elements

Videos are a dynamic tool that stimulates the visual and auditory channels of students, actively working on the Working Memory, which is limited in capacity. If the goal is to produce a video for educational purposes, many aspects must be taken care of, since to obtain the desired positive effect will depend on certain key elements (summarized in Table I-1).

Duration, form of narration, dynamism in the content and edition are what we conclude as essential in our case, and which we considered when we were preparing our video. But we have also understood that our production will be incomplete without a supplement like the one suggested in Active Learning (See Chapter 2.2.3). Nevertheless, we want to provide a tool that helps the field and that can be use by instructors.

Table I-1: Practices to maximize student learning from educational videos

Element to consider	Recommendation
Cognitive load	Use signaling to highlight important information.
	Match modality by using auditory and visual channels to convey complementary information.
Student engagement	Keep each video brief.
	Use conversational language.
	Speak relatively quickly and with enthusiasm.
Active Learning	Interactive Questions, Interactive Features, Guiding Questions or Make Video Part of a Larger Homework Assignment.

(Adapted from “Effective Educational Videos: Principles and Guidelines for Maximizing Student Learning from Video Content”, Brame, C. (2016), CBE—Life Sciences Education, 15:es6, page 3)

1.3. Estructure

Established the base knowledge for the reader's understanding, we will continue with the structure in which this thesis will be presented. As we mentioned previously in the beginning of the chapter, this thesis will examine the way in which a video can help in the students understanding of the Multinomial Logit Model (from now on MNL), based on previous studies. The reason to choose this topic for the video was based in personal interest after reading different sources and discover its broad utility and application for any aspiring administrator, as it is highly related to planning and forecasting consumer behavior tasks.

Throughout this thesis, the term Traditional Education, Virtual Education and Hybrid Education will be referred as TE, VE and HE, respectively. While Discreate Choice Model will be referred as DCM, Multinomial Logit Model as MNL, Independence of Irrelevant Alternatives as IIA and the Decision Maker will be represented as DM.

The overall structure will take the form of six chapters, including this instroductory section. In the second chapter everything related to the Educational Videos will be discussed: from the classification, the components that make them effective elements, to the types of education and how Traditional Education can be related to modern technology.

In chapters three and four we will discuss the theoretical framework that will serve as the basis for our audiovisual production. Starting with the DCM, the Utility Function, the basis for Discrete Models, and its elements. Moving on to the MNL, its properties, the probability function and the IIA, where we will touch an example treated in numerous studies: The Blue Bus/Red Bus paradox.

Everything related to the planning, script and development of the video will be discussed in chapter five. From the analysis of the possible editing programs, to how a final product was achieved. Finally, in chapter six a conclusion is presented on how the research of educational videos were related to our production project, and how we hope it will influence the future.

II. EDUCATIONAL VIDEOS

The year 2020 has put us all in the face of an alarming global crisis. It was a situation that no one could have been prepared for or could have imagined going through. With global isolation and quarantines in place to prevent an even more alarming situation, institutions and governments had to find ways to replace the way of working, without having to suspend everything until the vaccine was developed. Education was something essential and could not be stopped indefinitely, so Virtual Education (VE) became a forced trend.

Most of the classes were videoconferences or recordings of the teacher explaining a certain topic. Creativity was put to play, and technological resources something everyone should be familiar with. Like many people, we also participated in these classes, but not being able to be faced with the professors sometimes limited our understanding of certain topics. To complement our study, the books were a great help, but YouTube also became a crucial tool.

The virtual lectures were not perfect, and it was not the fault of the educators (forced to change their structures) that they were, but it was evident that a complement was necessary. It was participating in this whole situation that led us to the expeculation of implementing Animated Videos in the classes to reinforce the understanding of an important topic.

2.1. Educational Video - Types

The Cambridge dictionary define video as “a series of recorded images which are shown on television or viewed on a screen” (Video, n.d.). Since our work is focused on audiovisual productions projected in class or used via internet, adapted to the new technologies, this means that we are talking about the production of a digital video. The expression is defined by the Collins dictionary as the “video output based on digital rather than analogue signals.” (Digital Video, n.d.)

These digital videos will be considered as educational if their purpose is to educate, ergo to give information or explanations on a certain topic. With modern technology and the various existing platforms, we consider it necessary to exposed what we think are the types of Educational Videos that can be found, based on a list proposed by Vancouver Island University (n.d.). The classification was based on the editing tool implemented, the recording medium which was used and the way in it was presented.

2.1.1. Screencasts Videos

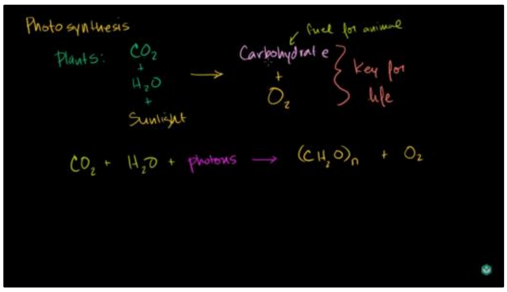
As the name implies, these videos are composed by the projection of a computer screen. According to Falaschi and Athey (2008) the Screencast Videos are composed with a screen or desktop adding the ingredient of movement on it (of at least eight frames per second), without still screen captues. Additionally, it must have an audio with "either a live narration of actions or a scripted voiceover of what is happening on screen."

The screencast expression came up on the blog of columnist Jon Udell (2004), after the contribution of readers who proposed various names for the development of, at that time, new technology. Some of the programs that allow the user to make this type of video are:

- Camtasia
- Screencast-O-Matic
- ScreenFlow
- Screencastify

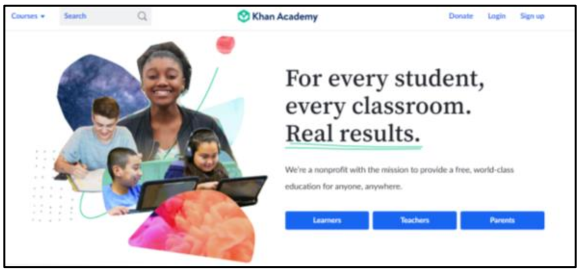
The Khan Academy is one of the most famous platforms at an educational level, which provides videos mainly in this format, intended to give tutoring on various educational topics. It was founden in 2008 by Salman Khan, an American educator, who sought to provide an online platform dedicated to education. According to Alexa ranking (a web traffic analysis), the website is ranked 470 (Alexa, n.d.) in the global traffic. The Khan Academy is a free website that allows to upload academic content, and provides a simple editing tool, which instructors can use to complement their classes. It is basically a black screen in which one can write, add images or other videos.

Figure II-1: Example Screencasts Video



(Screenshot from "Photosynthesis Video", of Khan Academy (2010). Retrieved from https://youtu.be/-rsYk4eCKnA)[1]

Figure II-2: Khan Academy website



(Screenshot from Kahn Academy website (n.d.). Retrived from https://en.khanacademy.org/)[1]

2.1.2. Talking-Head Video

This informal term is used to name a kind of video where the presenter or tutor talks directly to the camera, as if he were personally addressing the audience. The shot is taken in a close-up of the face or in a medium closeup. This type of video must take care of a series of factors to achieve a neat result: good lighting, clear sound, charismatic presenter (able to capture the attention of the audience), and a good, stable camera.

[1] Screenshot made on 08/09/20

Figure II-3: Example Talking-Head Video

(Screenshot from “20 German words AMERICANS USE all the time!”, of Hofner, F. (2020). Retrieved from https://youtu.be/aZoThD8NIjU)[1]

This type of filming is common to find in, for example, the news. It is also one of the most used nowadays by YouTubers in videos that range from tutorials to reviews. The YouTubers (wich is a person who uploads, produces, or appears in videos on the video-sharing website YouTube (YouTuber, n.d.)) were the ones who made it trending among teenagers.

2.1.3. Lecture/Classroom Recording

This was one of the most used tools in 2020 due to the pandemic. Since contact was restricted, many of the institutions transformed their lessons into virtual classes, and the Lecture Recording became almost something basic to every instructor. The concept is simple, it is about filming a lesson as if it were being presented in a traditional class-room, using boards or power point as support material.

The difference between Lecture recording and Classroom recording is simply the space. The first can be recorded in any environment, while the second is recorded in a classroom and originally it was simply a matter of recording the class in real time, with the students present. Due to the popularity it obtained, the number of tutorials to prepared one of these videos increased considerably during 2020, providing guides of which were the necessary elements to prepare one of these videos and the recommendations to achieve a final product with good quality.

This type of video requires a mixture of tools, both software and hardware, for its elaboration. For the physical part, it could be said that the same elements as for the videos of the Talking-Head Video are necessary (see example in Figure II-4): camera, light and microphone. While in the software part a program is necessary to make visual recordings, record sound and later allow a subsequent editing on the video. Genrally any advanced video editor covers all these functions.

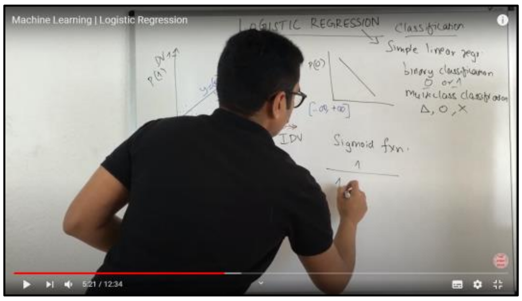
Just to name few examples of the tools that can be used specially to record lessons we can mention:

- Edmodo
- Moodle
- Schoology
- Panopto
- iSpring

Figure II-4: Elements to prepared a lesson and record it

(Reprinted from “5 steps to creating the ultimate lecture recording studio”, of Chernova, M. (2019). Retrieved from https://www.epiphan.com/blog/lecture-recording-studio/)

Figure II-5: Example of Lecture/Classroom Recording



(Screenshot from “Machine Learning | Logistic Regression”, of Ranji, R. (2020). Retrieved from https://youtu.be/S4IG2wv9Lnk)[2]

[2] Screenshot made on 08/09/20

2.1.4. Simulation Video

These videos are created to support the practical part of a lesson, in an attempt to help understand how the execution of certain situations are carried out. The main idea is to record a representation of a specific situation, which can occur (hence the name simulation). Visualizing the procedure makes it easier for the audience to copy and imitate what has been learned. It also helps to recognize patterns that can condition or even change the development of an escenario.

Generally, these types of videos are mostly used in medicine, biology, or mechanics, but its implementation is not ruled out in other areas.

Figure II-6: Example Simulation Video

(Screenshot from "Post-Partum Hemorrhage Simulation- Nursing Education", of Prof Tech Education- Peninsula College (2017). Retrieved from https://youtu.be/jorogc1urbk)[3]

2.1.5. Animation Video

Animation is defined as: "Moving images created from drawings, models, etc. that are photographed or created by a computer." (Animation, n.d.). That is, it is a representation of reality through a graphic creation which will depend on the number of images used to reproduce in a certain amount of time (frames per minute).

What distinguishes animations from the rest of the videos is that they are colorful and eye-catching, capable to cover a wide age range among their audience. In the past it took

[3] Screenshot made on 08/09/20

a long time to make an animation, since the drawings had to be made via hand frame by frame and then recompiled all in a single fluid movement. But with technological development, digital animation facilitated the creation of them. Among the types of animations that can be found, we distinguish the following:

1. Traditional animation: Each frame is drawn by hand.

Figure II-7: Traditional animation example

("Robin Hood", of Reitherman, W. (1973), Walt Disney Productions. Retrieved from https://i.imgur.com/T8zETAU.gif)

2. 2D animation: Vector-based animation in which movement is created in a two-dimensional space.

Figure II-8: 2D animation example

("Aladdin", of Clements and Musker (1992), Walt Disney Productions. Retrieved from https://giphy.com/gifs/disney-aladdin-genie-diamond-edition-A0I0ToQj8Fxja)

3. 3D animation: Computer animation where the body of the character's is completely visible.

Figure II-9: 3D animation example

("Tangled", of Conli, R. (2010), Walt Disney Productions. Retrieved from https://giphy.com/gifs/disney-tangled-rapunzel-pascal-NOnEwj4xBaPbG)

4. Motion graphics: Footage animated in a way that appears to be moving.

Figure II-10: Motion graphics example

(Reprinted from “WHY Motion Design & Motion Graphics WORK”, of Darvideo (n.d.). Retrieved from https://darvideo.tv/motion-design/)

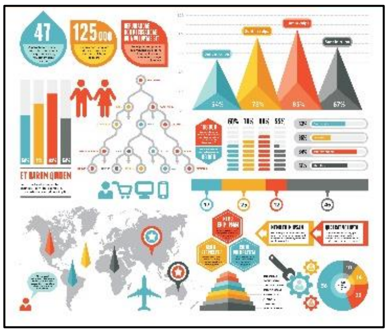
5. Whiteboard animation: Animation made with drawings on a white board, in which the audience can see how a hand makes the illustrations.

Figure II-11: Whiteboard animation example

(Reprinted from “Content creation for digital marketing”, of Russ Law (n.d.). Retrieved from https://www.russlaw.co.uk/explainer-video-services/content-creation/)

6. Infographic animation: As the name implies, it is an animation in which the information is displaced on a graph.

Figure II-12: Infographic animation example



(Reprinted from “Infographics are Dead. Long Live Infographics!”, of Taylor, L. (n.d.). Retrieved from https://lorirtaylor.com/infographics/)

7. Stop-motion animation: Technique in which an object is manipulated while taking pictures frame by frame to simulate a movement.

Figure II-13: Stop-motion animation example

(“Corpse Bride”, of Burton T. and Abbate A. (2005). Retrueved from https://www.awn.com/vfx-world/corpse-bride-stop-motion-goes-digital)

The choice of the type of animation wanted will condition the program that will be employed to design and edit the video. In the market the offer of these programs is very complete, getting to cover all types of animation. There are products of different prices ranges, to even those for free. Just to name some of them, the most recognized programs are: Adobe Character Animator, Cartoon Animator 4, Pencil2D and Moho (Anime Studio).

2.2. Accurate Content

This will be highly related to the objective for which we want to use the video (as we have mentioned before). If the content is not precise, the explanation may become ambiguous and produce an effect contrary to the desired. In their publication, Brame and Perez (2016) point out three elements that they consider fundamental when we are developing audiovisual content in orden to help the instructors:

a. Cognitive load

b. Student engagement

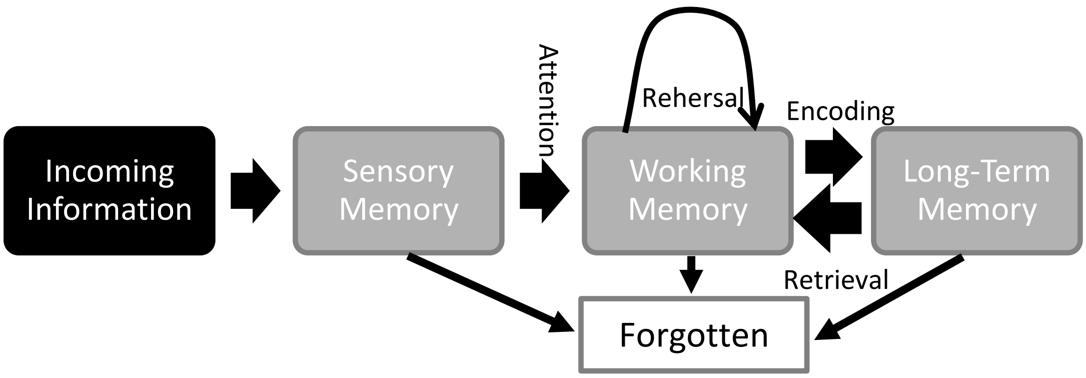
c. Active learning

"Together, these elements provide a solid base for the development and use of video as an effective educational tool." (Brame, 2016)

2.2.1. Cognitive Load

The Cognitive Load is a theory that seeks to give a scientific understanding of how the human brain works, in order to help develop educational material. In this field the Modal Model (Figure II-14) is the starting point, since it is a representation of the way memory works.

Figure II-14: Modal Model



(Adapted from “Human Memory: A Proposed System and its Control Processes”, by Atkinson R. and Shiffrin R., (1968))

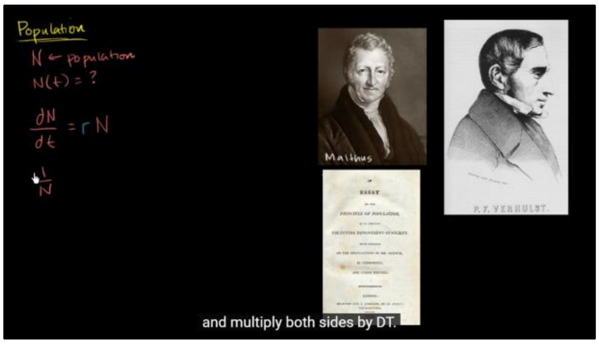
This model separates memory into the Sensory Memory, Working Memory and Long-Term Memory. Sensory Memory is the memory related to our senses and it is the first to receive the incoming information, it has limited storage, so it easily forgets its content. The next stage, called Working Memory, requires paying a certain degree of attention to the information captured by the Sensory Memory to enter the information to a reasoning phase. The information is stored for a longer time than the previous stage and can be remembered. Finally, when the information is reasoned and encoded, it is that it has access to Long-Term Memory, with a greater storage capacity than its predecessors and only forgettable after a long time.

John Sweller (1988) develops his theory of Cognitive Load to understand what is the amount of information that a person can perform into the entire process of the Modal Model in a given amount of time. For instructors, the type of memory to which they must focus is the Working Memory, since it is the one that is achieved if they capture the student's attention, but given that it is limited, the amount of information and the way in which it is presents must be taken care of.

This type of memory works with stimuli in two channels: the auditory and/or the visual. Each of these channels has a limited capacity, but the result can be improved with a good combination of both, as Brame and Perez explains: “Using both channels maximizes working memory’s capacity—but either channel can be overwhelmed by high cognitive load." (2016). Although the form and quantity of information presented must be taken care of, those statements also reinforce the theory that the visual complement in an explanation improves understanding and remembrance.

For example, a common form of teaching used during isolation was the online class with the teacher using a webcam so that his students could see him while he was speaking (a mix between Lecture Recording and Talking-Head Video), but this cannot be considered to cover both channels, since the visual part was not related with the content exposed, and therefore it did not meet our objective. To stimulate the visual channel, in a way that captures attention and it helps to process the information, other types of videos should be used, like the Screencasts Videos (such as the Khan Academy – style tutorial, see Figure II-15), the Simulation Videos or the Animation Videos.

Figure II-15: Khan Academy - style tutorial example



(Screenshot from “Modeling population as an exponential function | First order differential equations”, by Khan Academy (2014). Retrieved from https://youtu.be/_JpS8k1a9yE)[4]

In summary, the objective of every instructor should be to enter to the Working Memory of the students, then reaching the Long-Term Memory is easier for the student through the study. To achieve this, it is recommended to use the visual and auditory channels. “Using both channels to convey appropriate and complementary information

[4] Screenshot made on 08/09/20

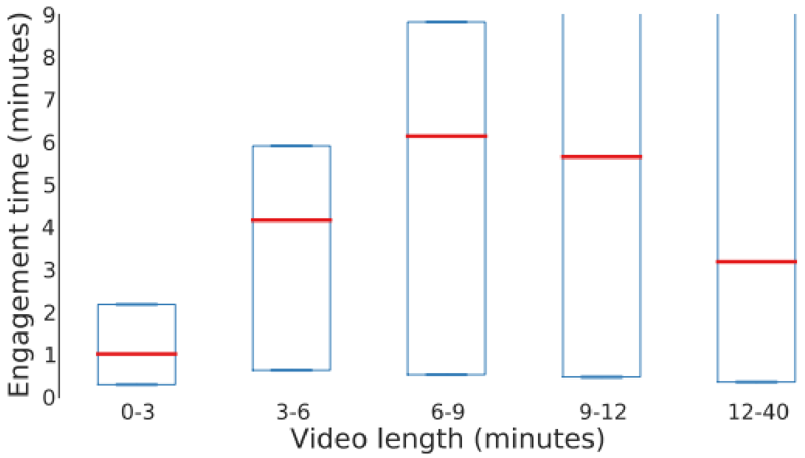
has been shown to increase students' retention and ability to transfer information and to increase student engagement with videos." (Brame, 2016)

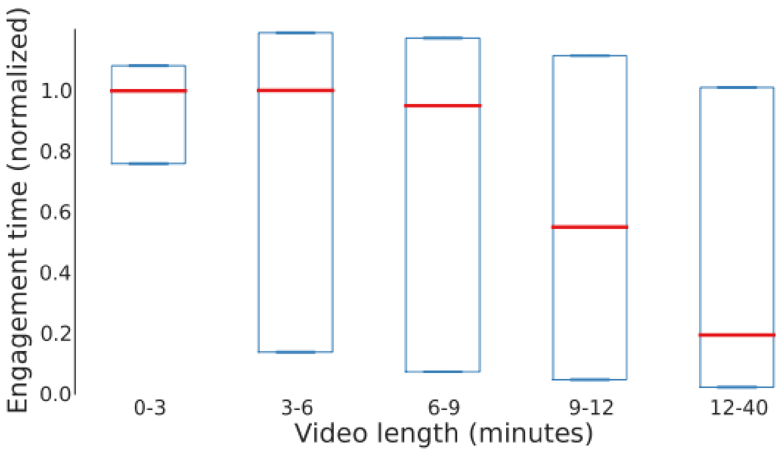
2.2.2. Student Engagement

To be effective with the audiovisual material, another element to consider is the attention of the students, also called the Student Engagement. The goal should be to capture it and keep it during an explanation, and for this the duration of the video should not exceed six minutes. This length comes from an investigation carried out by Gou, Kim and Rubin (2014), who used data from 6.9 million video watching sessions, trying to measure student engagement. The mean was represented in the graphs by a red line (see Figure II-16), and these show that the highest attention achieved during the experiment was six minutes.

They research also has shown that exceptionally long videos (considered as such to those videos with a duration greater than nine minutes) tended to lose the audience attention more quickly. On the other hand, videos of up to three minutes in length presented less variance than the rest, and the greatest attention from the receiver.

Figure II-16: Boxplots of engagement times in minutes





(Reprinted from “How video production affects student engagement: An empirical study of MOOC videos” Guo P., by Kim J. and Rubin R. (2014), Proceedings of the first ACM conference on Learning, p.44)

But only the duration is not enough to achieve engagement, vocabulary and voice color are other important tools. When we talk about voice color, we refer to the tone used to speak, and the emotions transmitted with it. This is not something new, since people tend to feel closer if the speaker us regular and warm ways to interact, while they tend to feel outside and away when the approach is formal. Mayer supports this theory in his article, presenting what he calls "The Personalization Principle" which states that: "(…) people learn better from a multimedia lesson when words are in conversational style rather than formal style." (2008)

Guo, Kim and Rubin (2014) also refers to this topic. In their research they proved that students preferred instructors who spoke fast, rather than those who spoke slowly or with many pauses. They hypothesized that this was due to the fact that: “(…) fast-speaking instructors conveyed more energy and enthusiasm(...)" (2014), wich reinforces once again our initial statment.

2.2.3. Active Learning

Although we have shown that videos are an advantageous tool for education, it is incomplete if it is not complemented with activities before or after its projection. These activities are meant to reinforce the content developed and assist encoding the information, helping it move from the "Working Memory" phase to the "Long-term Memory". There are many possible activities for this stage, and some of them were named by Brame and Perez (2016) in their article, such as:

- Interactive questions
- Interactive features
- Guiding questions
- Make a video as part of a larger homework assignment

2.3. Effectiveness of Educational Videos: Hybrid Education

Education can be classified according to the format in which a lesson is given, being possible to distinguish between: Tradional, Virtual or Hybrid Education. The first one, also known as conventional education, would represent face-to-face lessons between an instructor and his students. The second would represent those classes that are taken completely through the internet, hence its name. And, finally, Hybrid Education (or blended learning) is the one that mixes the other two.

The main idea of applying this type of education is to make use of the resources and benefits of each type of education, complementing face-to-face lectures with multimedia content accessible through internet. This is done with the goal of improve learning and achieve a better result than the one that would be achieved if only one of the other types of education were used.

Although many academics recommend it, a consensus on the matter has not yet been reached, so there is no perfect mix of how much to apply of one or the other. Yigit and colleagues (2014) addresses this issue in his article, stating that “there are many ways of applying blended learning. Therefore, there are no certain rules to define what the ideal blend might be." That is why it can be concluded that the way of use will be conditioned to the objective that the instructor wants to achieve.

For a better understanding of the difference between the diverse types of education, we set out their key points in a table:

Table II-1: Differences between TE, VE and HE

Main Features of Education	Traditional Education (TE)	Virtual Education (VE)	Hybrid Education (HE)
Location	In Physical Classes (Not Flexible)	Anywhere (Flexible)	In Physical Classes and Anywhere (Mixed Flexibility)
Learning Method	Face-to-Face	Online	Face-to-Face and Online
Learning Time	At Specific Time (Not Flexible)	Any Time (Flexible)	At Specific Time and at any Time (Mixed Flexibility)
Technology Usage	No obligation for using the Technology	It is a necessity to use the technology	It is a necessity to use the technology

(Adapted from Table 1 of "Evaluation of Blended Learning Approach in Computer Engineering Education", by Yigit T., Koyun A., Cankaya I. and Yuksel A. (2014), Procedia - Social and Behavioral Sciences, p.808)[5]

With a HE the elements of VE and TE are combined, managing to cover the weaknesses that each education style could present and making a better use of the resources. A highly recommended practice to implement HE could be giving traditional lectures in a classroom and later reinforce topics or key points with online information, especially with videos. The videos should develop definition or offer practical examples to improve understanding in students.

According to researchs conducted in the past, this method has proven to be the most efficient between the types of education. Among the investigations carried out we can name the one made by Means and colleagues (2009), whom compared studies carried out between 1996 to 2008 about the results obtained with TE and VE, and they discovered that online education (especially when it was done in a hybrid way), yielded a better result.

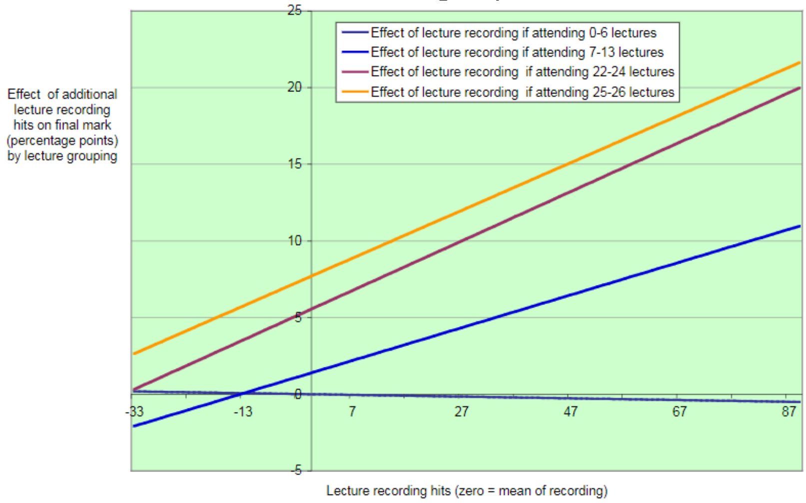
Martin and others (1995) also tested the effect of using videos, workbooks and computer packages on accounting students, wich result in increase of critical and reflective learning in students. Complementing the skills that a student can achieve, Rackway

[5] The names Traditional Learning and Blended Learnind were modified by Traditional Education and Hybrid Education respectively, to have a correlation with the names used at the beginning of the section. Also, the features of Virtual Education were added.

(2012) concludes that multimedia supplements improve learning in general, and Stockwell and others (2015) proved that videos could increased attendance and satisfaction in the students(we have referred to this previously as Student Engament, see 2.2.2). In a more recent study, Zhong (2017) tested the result obtained by instructors who produce educational videos, finding that "strong correlations were shown between student satisfaction and the instructor’s teaching effectiveness", which demonstrate, as in all the cases mentioned, the positive relationship that occurs between these two if multimedia media is used to enhance it.

Another crucial point that researchers agree is that the use of VE is efficient if it is used as a complement and not as a substitute for TE. This stasment was proved by Williams and others (2012) when they evaluated the performance of students against three scenarios: attending only to traditional lectures, viewing the lectures online, or attenging to lectures and later using the recorded material. Given this, they observed that those who made use of a single tool presented a lower performance than those who attended classes and used the videos after them (see Figure II-17). In other words, audiovisual material does not present any benefit if it is used alone, and not as a complement to a face-to-face lesson.

Figure II-17: Visual demostration of the effect on final marks of lecture recording use, by lecture attendance groups



(Reprinted from " The impact of online lecture recordings on studentperformance", by Williams A., Birch E. and Hancock P. (2012), Australasian Journal ofEducational Technology, p.209)

Llamas-Nistal and Mikic-Fonte (2014) carried out a similar test and they examinated the percentage of passed students from one lecture regarding the amount of time that these students accessed to the online material. The results again showed a clear improvement in those who used the material in a complementary way, respect to those who only used one resource.

With all these studies (and taking into account that there is much more that was left aside for a space issue), it is proven that HE is the effective result of mixing TE with VE, and must be managed as a set in order to obtain the most optimal results for students. Only one should not be used, leaving out the other. The tools are effective if they are used in a complementary way. As it has been summarized by Zupancic and Horz, "lecture recording is a very good supplement, but should not replace the live presentation." (2002)

III. DISCRETE CHOICE MODEL (DCM)

This model is used when we have a subject (the Decision Maker), who must choose between a finite number of alternatives (since it is discrete model), to help calculate the probability that each option will be chosen. The calculation will be conditioned to a series of elements, which are the following:

- The Decision Maker (DM)
- The Aternatives
- The Attributes of the Alternatives
- The Decision Rule

These elements are related to the decision-making process, which starts with the DM, who determines the available alternatives, continuing to appreciate the attributes of each alternative, to finally make a decision according to the Decision Rule (Ben-Akiva & Lerman, 1985). Already having clear how the elements are related, we proceed to explain each of them below.

3.1. DCM - Elements

3.1.1. The Decision Maker (DM)

Is the one that will execute the decision-making action, and it can be either a person or an organization. According with Koppelman and Bhat (2006) “The decision maker will depend on the specific choice situation.” For example, a person may need to decide where to buy a new cellphone, having as options a local in their city or buy it in an online store, while an organization should choose which channel use to offer their products. Both decisions will be conditioned by the attributes of each option, but in turn they are also conditioned by knowing how to get access to each of them, the knowledge to manage them, the confidence and security that each opption offers, and so on. That is, the DM will be limited in his choice, driven by his tastes and preferences, we call all this as individual characteristics of the DM.

These characteristics are different depending on the individual, so they must be taken into account in choosing the model, as Koppelman and Bhat (2006) states: “(…)it is important to develop choice models at the level of the decision maker and to include variables which represent differences among the decision makers.”

3.1.2. The Alternatives

These are given by the series of options that the DM considers in his decision-making process and they will be conditioned by the environment of the individual. The set of all the alternatives will part of what is known as the Choice Set. Hall (2003) explains that the said set is composed of all those potential alternatives, that is, for the analysis to be more complete, all available options are taken into account, even if they were chosen at the end, as if not. Unavailable alternatives will not be considered.

The number of alternatives that are part of the choice set must be finite, since we are talking about a deterministic model, and they must essentialy be feasible for the individual. Continuing with the previous example, buying a cell phone online would be ruled out if the subject did not have internet access. “The subset of the universal choice set that is feasible for an individual is defined as the feasible choice set for that individual.” (Koppelman & Bhat, 2006)

Summarizing, the sum of the alternatives will make up what is known as the Choice Set, which, according to Train (2009), must have 3 characteristics:

1. The alternatives must be mutually exclusive from the Decision Maker’s perspective, this means the DM cannot choose more than one alternative, therefore choosing one implies that the rest cannot be chosen.

2. The Choice Set must be exhaustive and should consider all possible options.
3. The number of alternatives must be finite, since we are talking about a deterministic model.

3.1.3. The Attributes of the Alternatives

They are formed by the particular characteristics of the different options. The importance of each attribute will be evaluated by the DM, who, in turn, will be the one who ranks it according to their preferences. If we use again the example of buying a new cell phone, some buyers will prefer a cheap price, others a high quality, and so on.

Attributes can be grouped, depending on its nature. “The attributes of alternatives may be generic (that is, they apply to all alternatives equally) or alternative-specific (they apply to one or a subset of alternatives).” (Koppelman & Bhat, 2006) It should also be taken in mind that we consider Attributes not only those that are quantifiable directly.

3.1.4. The Decision Rule

This is the name of the process that the DM goes through until making an election. Several elements are involved in the election, from rational to irrational. Several times the subject rationalize a choice, carefully considering the options with their positive and negative aspects, other times they choose on impulse, or do not think too much about their choice. Generally everyday choices are the ones we will find in the second group, while more important decisions (like buying a car) will be found in the first group.

We can also find situations in which individuals simply "follow the leader" in their choice, and copy it, which could be considered as part of irrational decisions, but this is not entirely true. Since if the leader made a previously thought choice, the follower would be copying his rational choice, skipping simply going through the process. (Koppelman & Bhat, 2006)

The decision made by the DM will be considered rational if it has the following characteristics:

- Is consistent, which means that the same election will always be chosen if the same situation occurs.
- It obeys the transitivity property, which refers to the fact that an element will be related in the same way compared to another element, always occupying the same position when an order is made.

Most models also consider the Utility Theory, as Hall (2003) explains in his book, the DM will be attracted by the value of a choice, which will lead him to select the option that presents the highest profit. This theory is considered robust by many theorists, and it will be used in this thesis as a basis, since “it provides a good description of the choice behavior even in cases where individuals use somewhat different decision rules.” (Koppelman & Bhat, 2006)

3.2. The Utility Theory

The objective of the DMC is not to determine the amount of resources that the DM required for each alternative, but it seeks to compute the number of times that a decision is made. One alternative is always preferred over the other, based on the principle that the individual will seek to maximize their utility. This is what is known as the Utility Theory.

These alternatives are chosen depending on other elements, which condition the decision-making process, such as the environment (that could be random and specific to each option) and will act as a constraint. The perceived utility, then, will always depend on the individual, his characteristics, and the Attributes of the Alternatives.

This is how we finally get to the utility function, which we will call U. Koppelman and Bhat (2006) stated that the mentioned function "has the property that an alternative is chosen if its utility is greater than the utility of all other alternatives in the individual's choice set." Expressed mathematically, this would be:

Equation III-1: The Utility Function U[6]

$$\text{If } U\ (X_i, S_n) \geq U\ \left(X_j, S_n\right) \forall\ j\ \Rightarrow i > j\ \ \forall\ j \neq i \in C$$

Where	U (…)	is the mathematical utility function,
	X_i, X_j	are vectors of attributes describing alternatives *i* and *j*, respectively,
	S_n	is a vector of characteristics describing individual *n*, that influence his/her prefereces among alternatives,
	$i > j$	means the alternative to the left is preferred to the alternative to the right, and
	$\forall\ j$	means all the cases *j,* in the choice set, and they are unequal to *i*.

From this formula we can interpret that, if alternative *i* is better than or equal to alternative *j*, it will be chosen from set C of options. This gives us the possibility to rank the total number of alternatives.

3.2.1. The Utility Function with ε

We assume that the DM has full capacity to prepare a ranked according to his convenience analysis of the alternatives given, but said analysis presents a lack of information (since it is impossible for a person to consider everything that exists in the universe of the set of alternatives), that is why, within the model, uncertainty must be included (Hall, 2003). According with Koppelman and Bhat (2006), "the data and models used by analysts describe preferences and choice in terms of probabilities of choosing each alternative rather than predicting that an individual will choose a particular mode with

[6]Adapted from "A Self Instructing Course in Mode Choice Modeling: Multinomial and Nested Logit Models", of Koppelman F. and Bhat C. (2006), U.S. Department of Transportation Federal Transit Administration, p.14. - Some letters of the notation were modified to give homogeneity in this thesis.

certainty." In other words, our model will reflect how likely it is that an individual will choose an alternative, considering both the known alternatives and those that they do not know, instead of reflecting how many decision makers actually chose that alternative.

Thus, modifying Equation III-1, the new equation will be formed as follows:

Equation III-2: The Utility Function with ε[7]

$$U_{in} = V_{in} + \varepsilon_{in}$$

Where	$\boldsymbol{U_{in}}$	is the true utility of the alternative *i* to the decision maker *n,*
	V_{in}	is the deterministic or observable portion of the utility estimated by the analyst, and
	ε_{in}	is the error or the portion of the utility unknown to the analyst.

The random variable represents the sum of possible errors in each scenario. They are unobserberd and unmeasured. As Train (2009) explains: "This decomposition is fully general, since ε_{in} is defined as simply the difference between true utility U_{in} and the part of utility that the researcher captures in V_{in}."[8] This error factor will be defined according with the researcher's criteria, and the representation of the choice situation that he considers appropriate for his study.

On the other hand, the deterministic factors are known and estimated. They will be developed deeply in the following sections

3.2.2. Components of the Utility Function

Using the Equation III-2 as a starting point, we can observe that this results from combining two components, the deterministic or observable portion (V_{in}), and the error or

[7] Adapted from "A Self Instructing Course in Mode Choice Modeling: Multinomial and Nested Logit Models", of Koppelman F. and Bhat C. (2006), U.S. Department of Transportation Federal Transit Administration, p.18. - Some letters of the notation were modified to give homogeneity in this thesis.

[8] This quote has been slightly modified in the letters used for mathematical notation, to give homogeneity in this thesis.

non-observable portion (ε_{in}). On the other hand, in the chapter 0 we developed the elements of the decision-making process, which were: The DM, the Alternatives, the Attributes of the Alternatives and the Decision Rule. Convining the two concepts, we can say that the last elements will influence the observable part in the formula, wich now will be formed by the characteristics of the DM, the Attributes of the Alternatives, and the combination of both factors. This relation it will be expressed mathematicaly as:

Equation III-3: The Deterministic term of the Utility Function (V.1)[9]

$$V_{in} = V(S_n) + V(X_i) + V(S_n, X_i)$$

Where	$\boldsymbol{V_{in}}$	is the deterministic portion of utility of alternative *i* for individual *n*,
	$V(S_n)$	is the portion of utility associated with characteristics of individual *n*,
	$V(X_i)$	is the utility of attributes for alternative *i*, and
	$V(S_n, X_i)$	is the portion of the utility which results from interactions between the attributes of alternative i and the characteristics of individual *n*.

In other bibliographies this equation can be found as follows:

Equation III-4: The deterministic term of the utility (V.2)[10]

$$V_{in} = V(z_{in}, S_n)$$

Where	$\boldsymbol{S_n}$	is like $\boldsymbol{V(S_n)}$ and is the vector of characteristics of individual *n,* and
	z_{in}	is the vector of attributes as perceived by individual *n* for alternative *i*.

Both formulas express the same thing, which would basically be the way the Decision Maker relates to the alternatives.

[9] Adapted from "A Self Instructing Course in Mode Choice Modeling: Multinomial and Nested Logit Models", of Koppelman F. and Bhat C. (2006), U.S. Department of Transportation Federal Transit Administration, p.19. - Some letters of the notation were modified to give homogeneity in this thesis.
[10]Reprinted from "Handbook of transportation science", of Hall, R. (2003), Kluwer Academic Publishers.

3.2.3. Attributes of Alternatives

The alternatives will be conditioned by their attributes (see section 3.1.3), which, to be considered in our formula, must be measurable and conditioner to the decision for DM. In a case of transportation, possible examples would be travel time, cost, alternatives, among others. The mathematical expression for $V(X_i)$, which is our factor assigned to the Attributes of the Alternatives that we observe in the Equation III-3, will be the following:

Equation III-5: Utility Associated with the Attributes of Alternatives[11]

$$V(X_i) = \gamma_1 \times X_{i1} + \gamma_2 \times X_{i2} + \cdots + \gamma_k \times X_{ik}$$

Where $\boldsymbol{\gamma_k}$ is the parameter which defines the effect of attribute *k* on the utility of an alternative, and

X_{ik} is the value of attribute *k* for alternative *i*.

We use factor γ_k to attribute the weight that each attribute will give to each alternative in the total sum.

3.2.4. Characteristics of the Decision Maker

Not all decisions made by the DM are logical, there are times when the subject makes choices based merely on personal preferences. These preferences are known as Bias, which result from multiplying β_i (the increase in the utility of alternative *i*) with ASC_i (equal to one for alternative *i* and zero for all other alternatives). According with Koppelman and Bhat (2006), “The differences in ‘bias’ across individuals can be represented by incorporating personal and household variables in mode choice models.”

[11] Reprinted from "A Self Instructing Course in Mode Choice Modeling: Multinomial and Nested Logit Models", of Koppelman F. and Bhat C. (2006), U.S. Department of Transportation Federal Transit Administration, p.20.

Therefore, we can define the formula of $V(S_n)$ as follows:

Equation III-6: Utility Associated with the Characteristic of the DM[12]

$$V(S_n) = \beta_{i0} \times ASC_i + \beta_{i1} \times S_{1t} + \beta_{i2} \times S_{2t} + \cdots + \beta_{im} \times S_{mn}$$

Where $\boldsymbol{\beta_{im}}$ is the parameter which defines the magnitude of the incremental bias du to an increase in the *m^th^* characteristic of the DM, and

S_{mt} is value of the *m^th^* characteristics for individual *n.*

3.2.5. The Error Term

As was mentioned at the beginning of the chapter, the ε_{in} element represents factors that will influence the utility funciton, and were not included in V_{in} (see Equation III-2). Due to its nature, the researcher cannot know its value, so, according to Train (2009), to help in its calculation the probability joint density of the vector is formed, which is expressed as follows:

Equation III-7: Alternative Choice Probability[13]

$$P_{in} = Prob\ (U_{in} > U_{jn}\ \forall\ j \neq i)$$

$$\Rightarrow P_{in} = Prob\ (V_{in} + \varepsilon_{in} > V_{jn} + \varepsilon_{jn}\ \forall\ j \neq i)$$

$$\Rightarrow P_{in} = Prob\ (\varepsilon_{jn} - \varepsilon_{in} < V_{in} - V_{jn}\ \forall\ j \neq i)$$

Where $\boldsymbol{P_{in}}$ is the probability that alternative *i* is chosen by DM *n*,

U (…) is the mathematical utility function,

[12] Adapted from "A Self Instructing Course in Mode Choice Modeling: Multinomial and Nested Logit Models", of Koppelman F. and Bhat C. (2006), U.S. Department of Transportation Federal Transit Administration, p.22. - Some letters of the notation were modified to give homogeneity in this thesis.
[13] Adapted from "Discrete Choice Methods with Simulation", of Train, K. (2009), Cambridge University Press, p.15. - Some letters of the notation were modified to give homogeneity in this thesis.

i and *j*	are describing alternatives *i* and *j*, respectively,
V	is the deterministic portion of the utility,
ε	is the error element of the utility,
$i > j$	means the alternative to the left is preferred to the alternative to the right, and
$\forall\, j$	means all the cases *j,* in the choice set, and they are unequal to *i*.

On this formula we will apply the cumulative distribution function in order to help us solving the probability, looking that the terms that represents the error elementes are below the observable terms. The Equation III-7 will be re-expressed as follows:

Equation III-8: Cumulative Distribution on ε[14]

$$\int_{\varepsilon} I\left(\varepsilon_{jn} - \varepsilon_{in} < V_{in} - V_{jn}\ \forall\, j \neq i\right) f(\varepsilon_n) d\varepsilon_n,$$

Where *I (…)* is the indicator function, equaling 1 when the expression in parentheses is true and 0 otherwise.

Train (2009) refers to *I* as "(…) a multidimensional integral over the density of the unobserved portion of utility, *f* (ε_n)." How this density is managed is what will determine the type of DCM that the researcher can implement.

3.3. Transportation Problem

For a better understanding of everything developed so far, we will apply the content in an example: The Transportation Problem. Let us start from the assumption that we want to know the preferences in the choice of transportation carried out by the population of a city, to which the following alternatives are presented:

[14] Adapted from "Discrete Choice Methods with Simulation", of Train, K. (2009), Cambridge University Press, p.15. - Some letters of the notation were modified to give homogeneity in this thesis.

Table III-1: Options Transportation Problem

Options	Where
Drive Alone (DA)	Represents the option of using a private transport to move a single individual.
Shared Ride (SR)	Represents the option of using private transportation to move more than one individual, who share the trip.
Transit (TR)	Represents the individual's option to use public transportation.

These alternatives (as we saw previously) have attributes, which will be considered in the decision-making process. For example: total travel time, in-vehicle travel time, out-of-vehicle travel time, travel cost, number of transfers, walk distance and reliability of on time arrival (Koppelman & Bhat, 2006). The mathematical expression of this could look like:

$$V(X_{DA}) = \gamma_1 \times TT_{DA} + \gamma_2 \times TC_{DA}$$ Equation III-9: Attributes of the DA[15]

$$V(X_{SR}) = \gamma_1 \times TT_{SR} + \gamma_2 \times TC_{SR}$$ Equation III-10: Attributes of the SR[15]

$$V(X_{TR}) = \gamma_1 \times TT_{TR} + \gamma_2 \times TC_{TR} + \gamma_3 \times FREQ_{TR}$$ Equation III-11: Attributes of the TR[15]

Where $\boldsymbol{TT_i}$ is the travel time for alternative *i*,

TC_i is the travel cost for alternative *i*,

$FREQ_{TR}$ is the frequency for transit services, and

γ_k is the parameter which defines the importance of the effect of each attribute *k*.

Cost and time will influence all the alternatives, while frequency is an attribute that will only be considered when we talk about public transport, for an obvious point that we do not depend on a frequency when we talk about DA and SR. Next, we will formulate the equation of the characteristics of the DM. For our example, we will consider the number of cars that a family owns, and the income of these, which will result in:

[15] Reprinted from "A Self Instructing Course in Mode Choice Modeling: Multinomial and Nested Logit Models", of Koppelman F. and Bhat C. (2006), U.S. Department of Transportation Federal Transit Administration, p.22.

$$V(S_{DA}) = \beta_{DA,0} \times 1 + \beta_{DA,1} \times Inc_n + \beta_{DA,2} \times NCar_n$$

Equation III-12: DM Characteristics for DA[16]

$$V(S_{SR}) = \beta_{SR,0} \times 1 + \beta_{SR,1} \times Inc_n + \beta_{SR,2} \times NCar_n$$

Equation III-13: DM Characteristics for SR[16]

$$V(S_{TR}) = \beta_{TR,0} \times 1 + \beta_{TR,1} \times Inc_n + \beta_{TR,2} \times NCar_n$$

Equation III-14: DM Characteristics for TR[16]

Where	$\boldsymbol{\beta_{i0}}$	is the modal bias constant for mode *i*,
	Inc_n	is the household income of the traveler *n*,
	$NCar_n$	is the number of cars in the traveler’s household, and
	β_{i1}, β_{i2}	are mode specific parameters on income and cars, respectively, for mode *i*.

Factor β is representing the measure of individual preference based on characteristics that the researcher cannot explain, and that are usually influenced by their personal characteristics (their background, their family, their nationality, among others).

On the other hand, we will have personal characteristics that will be influenced by the Attributes of the Alternatives and will differ depending on the variable in which it is analyzed. For example, a person with a higher income is going to show a preference in the DA option over the TR option. Also, we can represent the influence of the increasing income as a reduction of the importance of the TC (Koppelman & Bhat, 2006).

Finally, ε is added to our formula, which, as explained in previous sections, will represent the part of the investigation that is stochastic, therefore it is not possible to measure or calculat. This is how the equation in our example is re-expressed as follows:

[16] Adapted from "A Self Instructing Course in Mode Choice Modeling: Multinomial and Nested Logit Models", of Koppelman F. and Bhat C. (2006), U.S. Department of Transportation Federal Transit Administration, p.23. - Some letters of the notation were modified to give homogeneity in this thesis.

$$U_{DA} = V(S_n) + V(X_{DA}) + V(S_n, X_{DA}) + \varepsilon_{DA}$$ Equation III-15: Utility equation for DA[17]

$$U_{SR} = V(S_n) + V(X_{SR}) + V(S_n, X_{SR}) + \varepsilon_{SR}$$ Equation III-16: Utility equation for SR[17]

$$U_{TR} = V(S_n) + V(X_{TR}) + V(S_n, X_{TR}) + \varepsilon_{TR}$$ Equation III-17: Utility equation for TR[17]

Where $\boldsymbol{V}(\ldots)$ are formed by the deterministic components of the utility, and

ε_i it is formed by the stochastic components of the utility.

It is this error term which will lead us to modeling a mathematical model with the purpose to determine how it will be distributed over the deterministic components, and the impact it will cause on them. One of these models, object of our study, is the Multinominal Logit Model (MNL), developed in the next chapter.

[17] Adapted from "A Self Instructing Course in Mode Choice Modeling: Multinomial and Nested Logit Models", of Koppelman F. and Bhat C. (2006), U.S. Department of Transportation Federal Transit Administration, p.24. - Some letters of the notation were modified to give homogeneity in this thesis.

IV. THE MULTINOMIAL LOGIT MODEL

Based on what was developed in III, the stochastic component (ε_{in}) is the one that will control how our formula will be expressed, and it is the one that will determine the model that a researcher will implement according to how it is distributed over the rest of the components. The object of our study belongs to the Logit Models, which are defined according to Wooldridge (2013) as: “A model for binary response where the response probability is the logit function evaluated at a linear function of the explanatory variables”.

We already defined ε_{in} as the result of the difference between U_{in} (the utility of the alternative *i* to the decision maker *n*) and V_{in}(the deterministi portion of the utility). Therefore, as Train (2009) explains, ε_{in} will depend on the researcher's specification of V_{in}, and will not be defined by the choice situation *per se*.

4.1. Logit Models

Luce (1959) was one of the first to introduce the Logit formula in his book “Individual choice behavior: a theoretical analysis”, which he obtained through assumptions about the properties of making elections, wich was called the Luce's choice. This axiom is part of the theory of probability and is strongly related to the independence from irrelevant alternatives (IIA) that will be developed later (see Chapter 4.5). Then Marschak (1960) confirmed the formula whenever it was applied to a context where the DM looks to maximize benefits. The Random Utility representation (see chapter 3.2) was perfected by Luce and Suppes (1965) based on the work of E. Holman and A. A. J. Marley, giving rise to the logit distribution. Later McFadden (2001), will rename the model as: Multinomial logit Model. He will also be the one that makes a great development on its application in the economy (1974), becoming it an valuable tool and benchmark for other researchers, and winning the nobel prize for it.

The logit model uses the so-called Gumbel distribution (see Figure IV-1), in which ε_{in} (which comes from Equation III-2) is distributed independently and identically according to the probability density of extreme value type (Train, 2009). It is done in this way for the computational advantages of this distribution, applied in a context where maximization is important. As a result, it is observed that:

$$f(\varepsilon_{in}) = e^{-\varepsilon_{in}} e^{-e^{-\varepsilon_{in}}}$$

Equation IV-1: Gumbel probability density funcion[18]

$$F\ (\varepsilon_{in}) = \ e^{-e^{-\varepsilon_{in}}}$$

Equation IV-2: Gumbel cumulative distribution function[18]

With these formulas it can be assumed that the variance of the distribution is as follows:

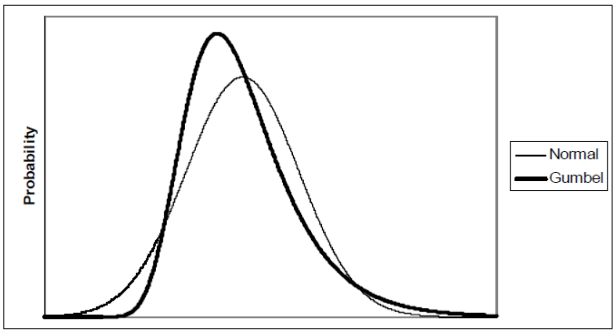
Equation IV-3: Variance of the distribution

$$Variance = \frac{\pi^2}{6\mu^2}$$

Where $\boldsymbol{\mu}$ is the scale parameter.

Assuming that this is the variance, the utility scale is implicitly normalized. The mean, in our study, is irrelevant since only the differences between profits are the ones that matter, and the difference between 2 values with the same mean has a mean equal to zero (Train, 2009).

Figure IV-1: Probaility Density Function for Gumbel and Normal Distributions (same mean and variance)



[18] Adapted from "Discrete Choice Methods with Simulation", of Train, K. (2009), Cambridge University Press, p.34. - Some letters of the notation were modified to give homogeneity in this thesis.

(Reprinted from "A Self Instructing Course in Mode Choice Modeling: Multinomial and Nested Logit Models", of Koppelman F. and Bhat C. (2006), U.S. Department of Transportation Federal Transit Administration, p.27)

The Multinomial Logit Model (MNL), object of our study, arises from three assumptions (Koppelman & Bhat, 2006):

1. ε_{in} is extreme-value distributed (Gumbel),
2. ε_{in} is identically and independently distribuited across alternatives, and
3. ε_{in} is identically and independently distributed across individuals.

In this model the probability of choosing an alternative *i* (*i* = 1,2,…,*j*) from a set of alternatives *J* is:

Equation IV-4: Probability MNL (V.1)[19]

$$P_{in} = Prob\ (V_{in} + \varepsilon_{in} > V_{jn} + \varepsilon_{jn}\ \forall\ j \neq i)$$

$$= Prob\ (\varepsilon_{jn} < \varepsilon_{in} + V_{in} - V_{jn}\ \forall\ j \neq i)$$

Where $\boldsymbol{P_{in}}$ is the probability of the DM choosing alternative *i,*

V_{xn} is alternative *i* and *j,* respectively, and

ε_{xn} is the error element.

Algebraic manipulations are applied to the equation, which will result in:

Equation IV-5: Probability MNL[20]

$$P_{in} = \frac{e^{V_{in}}}{\sum_{j=1}^{J} e^{V_{jn}}}$$

Where $\boldsymbol{P_{in}}$ is the probability of the DM choosing alternative *i*,

[19] Adapted from "Discrete Choice Methods with Simulation", of Train, K. (2009), Cambridge University Press, p.36. - Some letters of the notation were modified to give homogeneity in this thesis.

[20] Adapted from "A Self Instructing Course in Mode Choice Modeling: Multinomial and Nested Logit Models", of Koppelman F. and Bhat C. (2006), U.S. Department of Transportation Federal Transit Administration, p.28. - Some letters of the notation were modified to give homogeneity in this thesis.

V_{in}	is the systematic component of the utility of alternative *i*,
V_{jn}	is the systematic component of the utility of alternative *j*, and
J	represents the Choices Set.

The probability in the Logit Model has several properties, among them we have that P_{in} take a value between zero and one. As Train (2009) explains in his book, when there is an increase in V_{in}, due to an improvement in the observed attributes of the alternative, and as long as the variables $V_{jn}\ \forall\ j \neq i$ are kept constant, P_{in} approaches one. Likewise, if the change occurred in V_{jn}, P_{in} would approach zero. What we have to keep in mind is that the Logit probability for is never equal to zero, since if the researcher thinks that an alternative has no chance of being chosen, they simply will not include it in the set of alternatives. On the other hand, if the probability is exactly equal to one, would reflect that the set of alternatives (or Choices Set) is made up of only one choice.

Another property of Logit probability is that the sum of all the probabilities of the options in the Choices Set is going to be equal to one, which represents that the DM is going to choose only one of the given alternatives.

4.2. Transportation problem with Logit Probability

If we wanted to apply what was developed in the Chapter 4.1 over the case of the transportation problem formulated in 3.3, it could be observed that the probabilities of the three alternatives proposed (DA, SR and TR) would be represented as follows:

$$P_{DA} = \frac{e^{V_{DA}}}{e^{V_{DA}} + e^{V_{SR}} + e^{V_{TR}}}$$

Equation IV-6: Probability for DA[21]

[21] Adapted from "A Self Instructing Course in Mode Choice Modeling: Multinomial and Nested Logit Models", of Koppelman F. and Bhat C. (2006), U.S. Department of Transportation Federal Transit Administration, p.29. - Some letters of the notation were modified to give homogeneity in this thesis.

$$P_{SR} = \frac{e^{V_{SR}}}{e^{V_{DA}} + e^{V_{SR}} + e^{V_{TR}}}$$

Equation IV-7: Probability for SR[21]

$$P_{TR} = \frac{e^{V_{TR}}}{e^{V_{DA}} + e^{V_{SR}} + e^{V_{TR}}}$$

Equation IV-8: Probability for TR[21]

Where	$\boldsymbol{P_{DA}}$, $\boldsymbol{P_{SR}}$ and $\boldsymbol{P_{TR}}$	are the probability of the DM choosing DA, SR and TR, respectively, and
	V_{DA}, V_{SR} and V_{TR}	are the systematic components of the utility for DA, SR and TR, respectively.

To arrive at this formula, the different alternatives were applied in our Equation IV-5. If we wanted to simplify Equation IV-6, Equation IV-7 and Equation IV-8 in a single general equation, it would be done as:

Equation IV-9: General probability equation for the transportation problem[22]

$$P_{in} = \frac{e^{V_{in}}}{\sum_{j=DA,SR,TR} e^{V_{jn}}}$$

Where	$\boldsymbol{i}$	is the alternative for which the probability is being computed.

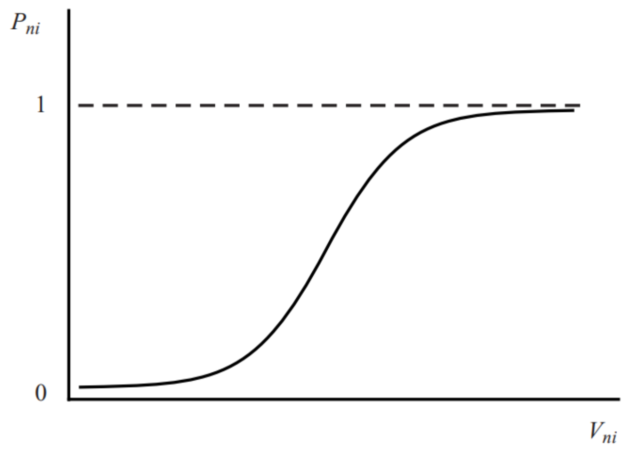
The equation reflects that the probability of a selected alternative will increase if the utility of that alternative increases and will decrease if the utility of another alternative increases.

4.3. The Sigmoid shape of Multinomial Logit Probabilities

[22] Adapted from "A Self Instructing Course in Mode Choice Modeling: Multinomial and Nested Logit Models", of Koppelman F. and Bhat C. (2006), U.S. Department of Transportation Federal Transit Administration, p.30. - Some letters of the notation were modified to give homogeneity in this thesis.

Due to the relationship of the utility of an alternative to the utility of all the alternatives, the graphical form obtained to represent it is an *S*, also known as Signmoid, as it is shown in Figure IV-2. In this graph you can see the limits for an alternative, which will be close to zero when the utility is low compared to the rest of the set of options, and close to one when the utility is high in the comparison.

Figure IV-2: The Sigmoid shape of Multinomial Logit Probabilities



(Reprinted from "Discrete Choice Methods with Simulation", of Train, K. (2009), Cambridge University Press, p.38)

Also this graph helps to show the degree of impact that the change in the variables can produce: if one alternative have low utility compared with the other alternatives, a small change in its utility will have little effect on the probability of being chosen. In the same way it happens if an alternative has a utility much higher than the rest in the Choices Set, an increase in its utility will have little effect on its probability of choice. This graph helps to identify the cases where the choice has a difference in the utility that is not so great, and a small change occurs.

4.4. The Equivalent Differences (ED) Property

One fundamental property in Logit Models is that the probabilities of an alternative only depends on the difference in the utilities of the rest component of the Choice Set, and not in their values. For a better understanding of this we will make use of our example already formulated in Transportation Problem (see Chapter 3.3). We will start from formulas Equation IV-6, Equation IV-7 and Equation IV-8, and we will assume that the utility of our three alternatives have the following values:

Table IV-1: Values for the Transportation Problem example

Drive Alone (DA)	-0.5
Shared Ride (SR)	-1.5
Transit (TR)	-3.0

Making the corresponding replacements, and calculating the result, we obtain that:

$$P_{DA} = \frac{e^{-0.5}}{e^{-0.5} + e^{-1.5} + e^{-3.0}} = 0.690$$

Equation IV-10: Probability for DA in ED example[23]

$$P_{SR} = \frac{e^{-1.5}}{e^{-0.5} + e^{-1.5} + e^{-3.0}} = 0.254$$

Equation IV-11: Probability for SR in ED example[23]

$$P_{TR} = \frac{e^{-3.0}}{e^{-0.5} + e^{-1.5} + e^{-3.0}} = 0.057$$

Equation IV-12: Probability for TR in ED example[23]

If these probabilities are increased by one, the equations would be formed as:

$$P_{DA} = \frac{e^{0.5}}{e^{0.5} + e^{-0.5} + e^{-2.0}} = 0.690$$

Equation IV-13: Probability for DA in ED example with increasement[23]

$$P_{SR} = \frac{e^{-0.5}}{e^{0.5} + e^{-0.5} + e^{-2.0}} = 0.254$$

Equation IV-14: Probability for SR in ED example with increasement[23]

$$P_{TR} = \frac{e^{-2.0}}{e^{0.5} + e^{-0.5} + e^{-2.0}} = 0.057$$

Equation IV-15: Probability for TR in ED example with increasement[23]

[23] Adapted from "A Self Instructing Course in Mode Choice Modeling: Multinomial and Nested Logit Models", of Koppelman F. and Bhat C. (2006), U.S. Department of Transportation Federal Transit Administration, p.33. - Some letters of the notation were modified to give homogeneity in this thesis.

This reflects that the probabilities are the same as those obtained previously, since the increase occurred in all the alternatives in the Choices Set. If we put the information in tables, they will be summarized as follows:

Table IV-2: Numerical example of ED Property: Probability of each Alternative before Delta[24]

Alternatives	Utility Expression	Utility Value	Exponent	Probability
Drive Alone	-0,50	-0,50	0,6065	0,6897
Shared Ride	-1,50	-1,50	0,2231	0,2537
Transit	-3,00	-3,00	0,0498	0,0566
		∑exp	0,8794	

Table IV-3:Numerical example of ED Property: Probability of each Alternative after adding Delta (=1,00)[24]

Alternatives	Utility Expression	Utility Value	Exponent	Probability
Drive Alone	-0,50+1,00	0,50	1,6487	0,6897
Shared Ride	-1,50+1,00	-0,50	0,6065	0,2537
Transit	-3,00+1,00	-2,00	0,1353	0,0566
		∑exp	2,3906	

Due to the property of the Equivalent Difference (ED), the probability formula for the Logit Models can also take the following form if we start from the Equation IV-5:

Equation IV-16: Binary Logit Probability MNL[25]

$$P_{in} = \frac{1}{1 + \sum_{j \neq i} e^{(V_{jn} - V_{in})}} \quad \forall\, i \in J$$

This equation is what is known as Binary Logit Probability, and is a result of multiplying the numerator and denominator of the Equation IV-5 by the numerator in negative, as it's shown in the following equation:

[24] Reprinted from "A Self Instructing Course in Mode Choice Modeling: Multinomial and Nested Logit Models", of Koppelman F. and Bhat C. (2006), U.S. Department of Transportation Federal Transit Administration, p.34

[25] Adapted from "A Self Instructing Course in Mode Choice Modeling: Multinomial and Nested Logit Models", of Koppelman F. and Bhat C. (2006), U.S. Department of Transportation Federal Transit Administration, p.35. - Some letters of the notation were modified to give homogeneity in this thesis.

Equation IV-17: Probability MNL multiplied by its negative numerator[25]

$$P_{in} = \frac{e^{V_{in}}}{\sum_{j=1}^{J} e^{V_{jn}}} \times \frac{e^{-V_{in}}}{e^{-V_{in}}}$$

This formula represents that the probability of an alternative *i* is a function of the difference in the utility of said alternative *i* and the rest of the alternatives in the Choices Set *J*. As an example, we will apply this formulation to our case on the transportation problem for the probability of Driving Alone (DA):

Equation IV-18: Example Binary Logit Probability MNL[25]

$$P_{DA} = \frac{e^{V_{DA}}}{e^{V_{DA}} + e^{V_{SR}} + e^{V_{TR}}} \times \frac{e^{-V_{DA}}}{e^{-V_{DA}}}$$

$$= \frac{e^{V_{DA}} \times e^{-V_{DA}}}{(e^{V_{DA}} + e^{V_{SR}} + e^{V_{TR}}) \times e^{-V_{DA}}}$$

$$= \frac{e^{0}}{e^{0} + (e^{V_{SR}} - e^{-V_{DA}}) + (e^{V_{TR}} - e^{-V_{DA}})}$$

This formula is simplified to:

Equation IV-19: Example Binary Logit Probability MNL Simplified[25]

$$P_{DA} = \frac{1}{1 + (e^{V_{SR}} - e^{-V_{DA}}) + (e^{V_{TR}} - e^{-V_{DA}})}$$

4.5. Independence of Irrelevant Alternatives Property

Finally, we arrive at the Independece of Irrelevant Alternatives Property (from now on IIA), strongly related to the paradox that we will try to explain in our audiovisual production. This property explains something that has already been mentioned previously in this chapter, which is that if an alternative improves, the probability of choosing said alternative increases. Logically this reflects that those DM who had chosed an alternative, after the variation in the attributes of the alternatives, they will tend to change their choices. As Train (2009) explains: “Since probabilities sum to one over alternatives, an increase in the probability of one alternative necessarily means a decrease in probability for other alternatives."

Now, for the MNL, the IIA expresses that the probability of choosing between two alternatives is independent of the attributes of other alternatives. This occurs since, according to what Koppelman and Bhat (2006), the premise is that if we consider the alternatives in pairs, other alternatives are irrelevant to the DM. "The independence assumption follows from the initial assumption that the disturbances are independent and homoscedastic" (Greene, 2008), understanding that homoscedasticity occurs when the variance of the conditional error to other variables remains contant throughout the observations.

To help understand this property, we will use an example. We will consider an MNL for a the transportation decision process among the alternatives of Car, Bus or Train, whose alternatives would be:

$$P_{Car} \frac{e^{V_{Car}}}{e^{V_{Car}} + e^{V_{Bus}} + e^{V_{Train}}}$$

Equation IV-20: Probability for IIA example: Car[26]

$$P_{Bus} = \frac{e^{V_{Bus}}}{e^{V_{Car}} + e^{V_{Bus}} + e^{V_{Train}}}$$

Equation IV-21: Probability for IIA example: Bus[26]

$$P_{Train} = \frac{e^{V_{Train}}}{e^{V_{Car}} + e^{V_{Bus}} + e^{V_{Train}}}$$

Equation IV-22: Probability for IIA example: Train[26]

We will try to consider the alternatives in pairs, to prove the property of the IIA, which will result in the following ratios:

$$\frac{P_{Car}}{P_{Bus}} = \frac{e^{V_{Car}}}{e^{V_{Bus}}} = e^{(V_{Car} - V_{Bus})}$$

Equation IV-23: Ratio for IIA example between Car and Bus[26]

$$\frac{P_{Car}}{P_{Train}} = \frac{e^{V_{Car}}}{e^{V_{Train}}} = e^{(V_{Car} - V_{Train})}$$

Equation IV-24: Ratio for IIA example between Car and Train[26]

[26] Reprinted from "A Self Instructing Course in Mode Choice Modeling: Multinomial and Nested Logit Models", of Koppelman F. and Bhat C. (2006), U.S. Department of Transportation Federal Transit Administration, p.39.

$$\frac{P_{Bus}}{P_{Train}} = \frac{e^{V_{Bus}}}{e^{V_{Train}}} = e^{(V_{Bus}-V_{Train})}$$

Equation IV-25: Ratio for IIA example between Bus and train[26]

The ratios of each pair of alternatives only depend on the alternatives involved and its attributes, regardless of the state of the third alternative, its availability or its attributes. This property is called IIA because the two alternatives considered in the ratio are considered without any influence from the other alternatives, which are called irrelevant alternatives. The general formula for calculating the ratio for pairs of alternatives (*i* and *k*) would be represented as:

Equation IV-26: Ratio for IIA[26]

$$\frac{P_i}{P_k} = \frac{e^{V_i}}{e^{V_k}} = e^{V_i-V_k}$$

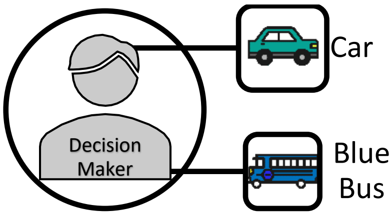
This property has advantages when, for example, the researcher wants to add or remove an alternative from the Choice Set, without causing an alteration in the structure or parameters of the model. Another advantage is that the model is applied in cases where DM face different alternatives, assisting in making easy the probabilities calculations. The disadvantage of its application is that it may not reflect the reality, and the alternatives left aside may not be irrelevant to the DM.

4.5.1. The Red Bus and Blue Bus Paradox

This paradox is used to explain situations in which the MNL can lead to an error in the calculation. We assume that we have a worker who must travel to work every day, and his transportation options are the following: Driving a Car or taking a Blue Bus (see

Figure IV-3).

Figure IV-3: IIA Paradox Example Situation 1



If we say that the utility of both alternatives is the same, the probability of each alternative will be, therefore, also the same. Expressed mathematically:

Equation IV-27: Probability of choosing Car and Blue Bus in IIA example[27]

$$P_C = P_{BB} = \frac{1}{2}$$

Where $\boldsymbol{P_C}$ represents the probability of the alternative of traveling by Car, and

P_{BB} represents the probability of the alternative of traveling by Blue Bus.

And the probability ratio (theoretical framework developed in Equation IV-26) would be:

Equation IV-28: Probability Ratio in IIA example[27]

$$\frac{P_c}{P_{BB}} = 1$$

Now, a new alternative is added to our scenario: A Red Bus. Which will have the same attributes as the alternative of the blue bus, so that the traveler (our DM) will consider them completely as equals. The situation has changed, which is reflected in the

Figure IV-4. As this new alternative is equivalent for the DM than the Blue Bus, the ratio of both probilities will be expressed as:

[27] Reprinted from "Discrete Choice Methods with Simulation", of Train, K. (2009), Cambridge University Press, p.46.

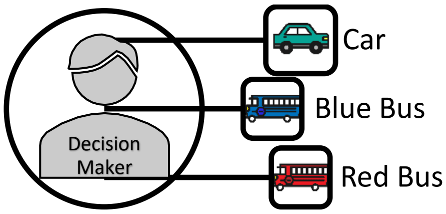
Equation IV-29: Probability Ratio Blue Bus and Red Bus[27]

$$\frac{P_{RB}}{P_{BB}} = 1$$

Where P_{RB} represents the probability of the alternative of traveling by Red Bus, and

P_{BB} represents the probability of the alternative of traveling by Blue Bus.

Figure IV-4: IIA Paradox Example Situation 2



The problem arises when we consider once more the first ratio (see Equation IV-28), which, according to IIA property, must remain the same as before than the new alternative of the red bus was incorporated (that is, the Equation IV-28 should continue to have a value equal to one). The only way that ratios in Equation IV-28 and

Equation IV-29 remain the same is that the probability of all alternatives is changed (as it is shown in Equation IV-30)

Equation IV-30: Value of the Probabilities of the Alternatives in the Paradox Example[27]

$$P_c = P_{BB} = P_{RB} = \frac{1}{3}$$

But assigning the same probabilistic value to the alternatives is far from reality, since, due to the attributes of the alternatives of the Red Bus and the Blue Bus, it would be expected that the probability that P_{BB} initially had would be divided between both alternatives, while the probability for Car (P_c) remains the same. Mathematically the scenario would be represented as:

Equation IV-31: Optimal value of the Probabilities of the Alternatives in the Paradox Example

$$P_c = \frac{1}{2}\ , and\ P_{BB} = P_{RB} = \frac{1}{4}$$

This reflects that in the Muntinomail Logit Model, due to the property of the IIA, situations such as this example can occur, where the probability of an alternatives could be overestimated (like the buses in our example), while other options (like the car) are underestimated in order to keep the indifference.

4.5.2. Solution to IIA Paradox

As we have developed in the previous section (see Chapter 4.5.1), the IIA property can lead to an error in the probability calculation for the MNL. Numerous researchers have done studies over this paradox like Cheng and Long (2007) or Zhang and Hoffman (1993), trying to understand the nature of the error, how the property works and looking for a solution. This is how we arrived at Müller and Haase investigation (2014).

Reviewing property IIA, it is shown that the probability of two alternatives remains constant regardless of the rest of the alternatives in the Choice Set and the state in which they are (whether they are available, or not). The problem comes when a new alternative is incorporated, which can lead to an error in the probability calculation if one tries to respect the IIA (as it was shown in 4.5.1). In their article, Haase and Müller concluded that the mentioned error can be reduced if the researcher applies a segmentation in his population. Said segmentation must be adequate: the more precise the segmentation, the lower the calculation error.

In order to make a correct segmentation, the researcher must know the attributes of the population under analysis, and should clearly specify the characteristics of the DM (mentioned in the chapter 3.2.4), this can facilitate a competent segmentation according to the research that is desired to be carried out.

If we express all this mathematically, it could be said that the way to reduce the error caused by the property of the IIA would be the following:

1. For V_{in} (originated in the Equation III-2) the focus will be done $V(S_n)$, which represents the portion of utility associated with characteristics of individual *n* (see Equation III-3), looking to make it as complete as possible to obtain a better result after applying MNL, and
2. If segmentation is applied, when we try to maximize the result of Equation IV-5 (Probability MNL), the calculation error for IIA can be reduced in Equation IV-26 (Ratio for IIA).

For a better understanding of how this works, we will use the example given in Müller and Haase (2014). We will consider a demand made up of two individuals, such that $n \in \{1,2\}$, where tree alternatives are given for the acquisition of a certain good, making $J = \{A, B, C\}$ (where *J* represents the Choice Set). The deterministic utility will be formed by:

Equation IV-32: The deterministic utility in example of Müller and Haase[28]

$$V_{in} = \frac{-g_i}{q_n} \ \forall\ i \in J$$

Where	$\boldsymbol{V_{in}}$	is the deterministic portion of utility of alternative *i* for individual *n*,
	g_i	represents the cost of the good in alternative *j*,
	q_n	represents the income of individual *n,* and
	$\forall\ i \in J$	means all the cases *i,* in the Choice Set *J*.

Next, two ways of calculating P_{in}(which is the probability of the DM *n* of choosing alternative *i,* see Equation III-5) are presented:

1. An average of the income of the two individuals is used, mathematically:

[28] Adapted from "Customer segmentation in retail facility location planning", of Müller S. And Haase K. (2014), Business Research, p.240. - Some letters of the notation were modified to give homogeneity in this thesis.

Equation IV-33: Formula for average of the income in the example[29]

$$\bar{q}_{i'} = \frac{(q_1 + q_2)}{2}$$

Used to calculate $\bar{P}_{in}$, or

2. The probability of choosing alternative *i* is calculated for each individual *n*, and then an average of the probabilities obtained in both individuals is made, such that:

Equation IV-34: Average of choice probability $\forall\, n$ in the example[29]

$$\tilde{P}_{in} = \frac{(P_{i,n=1} + P_{i,n=2})}{2}$$

Because of the non-linear relationship between P_{in} and V_{in}, alternative 1 is expected to be inaccurate compared to alternative 2, this is demonstrated in

Table IV-4. In it, two solutions will be presented: Solution 1, which will consider the entire Choice Set *J*, and Solution 2, which will leave out of consideration the alternative *j* = B.

Table IV-4: Values and calculations for the example[29]

	j = A	j = B	j = C	
g_i	1	2	4	
q_1	9	9	9	
q_2	1	1	1	
$\bar{q}_{i'}$	5	5	5	
Solution 1	**1**	**1**	**1**	$P_{i=A,n} / P_{i=C,n}$
$P_{i,n=1}$	0.383	0.343	0.274	1.396
$P_{i,n=2}$	0.705	0.259	0.035	20.085
$\tilde{P}_{in}$	0.544	0.301	0.155	**3.516**
P_{in} using $\bar{q}_{i'}$	0.422	0.346	0.232	1.822

[29] Adapted from "Customer segmentation in retail facility location planning", of Müller S. And Haase K. (2014), Business Research, p.240. - Some letters of the notation were modified to give homogeneity in this thesis.

Solution 2	**1**	0	**1**	
$P_{i,n=1}$	0.583	0	0.417	1.396
$P_{i,n=2}$	0.953	0	0.047	20.085
$\tilde{P}_{in}$	0.768	0	0.232	**3.302**
P_{in} using $\bar{q}_{i'}$	0.646	0	0.354	1.822

The last column represents a comparison similar to that of the Equation IV-26, which reflect the property of IIA. As can be seen in the results, the property remains accurate when each individual *n* is considered separately, and not when it is considered in the whole population (which we will call *n'*). As Müller and Haase (2014) explain: "The key point is that there are two distinct segments if the population (high and low income) with different choice probabilities", this probability will be affected according to the impact that an economic change represents to each category. The individuals at $n = 2$ will have a greater reaction to a price change with respect to the individuals at $n = 1$.

All this serves to demonstrate what has been previously developed: That segmentation helps mitigate the calculation error for the MNL due to IIA. The way in which such segmentation will be done, its complexity and the parameters to be considered will always depend on the nature of the investigation and each particular scenario.

4.5.3. IIA advantages

Despite its flaws and ineffectiveness in certain circumstances (see Chapter 4.5.1 for an example), the model is a complete tool and can be used to facilitate calculations thanks to the IIA property. Given that one alternative is considered indifferent to another, this can facilitate the researcher to simplify the number of alternatives to be investigated, which would result in saving energy, effort, and money. For example, let us continue with our transportation problem and suppose that a researcher wants to find out how the population goes from home to work. The options are numerous: walking, by bicycle, bus, tram, train, car, motorcycle, among others. Thanks to the IIA, the researcher can only choose to test five options from a long list, and it will be considered representative of reality, since the probability of these five alternatives is indifferent to whether there are other alternatives.

The property also facilitates the incorporation of new alternatives to the analysis if it where necessary and would not alter the probabilities already calculated previously. All this represents energy and time savings for the researcher. On the other hand, if the researcher worked, for example, in the public transport planning department, and he would like to calculate the probability of the population's choice only on those means handled by public service (tran, train or bus), he will be able to remove the rest of the alternatives from his analysis, and this will remain being a reflection of reality.

In short, thanks to the IIA, researchers are capable of make a better use of they time, since it allows to simplify the work. Used when the number of alternatives is exceptionally large and difficult to calculate, to when a special focus is required on a certain Choice Set, avoiding the alternatives that do not concern a particular investigation. All this without affecting the probability calculation and keeping it accurate.

V. MNL VIDEO PRODUCTION

In the previous chapters we have understood about the importance of Hybrid Education, and how effective it can be when is implemented as a complement to Traditional Education through the use of videos that reinforce knowledge (see Chapter 2.3). With the aim of helping the econometrics students, we started our audiovisual production project.

The basic starting point was to make a summary, pointing the essential topics of the theme that we wanted to develop in the video, which in our case was the Multinomial Logit Model (see IV). We wanted our video to be a support to reinforce the understanding specifically about the IIA, that is why we planned it from a hypothetical situation, easy for viewers to understand.

From existing types of Educational Videos (which were developed in the II), it was decided to opt for an Animated Video, and within these a Whiteboard was chosen (see 0). The choice was made based on the fact that:

- The animation can capture the student's attention with less effort than the other types of video
- The white background and simple drawings do not exhaust the user with overloaded screens
- With the student's attention and clear and simple scenes, the Student Engagement (see Chapter 2.2.2) can be achieved
- Is a tool that gets positive responses

Made the choice, we proceeded to the investigation of possible alternatives for editors and design, which we will develop below.

5.1. Video Editors

To create our Whiteboard Video, we were faced with two options: we could shoot the video by hand, with the use of a physical whiteboard, or we could used one of the animated editors offered on the internet.

The option of making the video by hand had its advantages, like, for example, let our imagination run free at the time of presenting our idea on the board, without being limited by the elements that a software editor offers. But it also had its disadvantages, especially in the technical part since recording a video by hand has different physical requirements: one needs a free space, the whiteboard, a good camera capable capturing the shots with clarity, and, most importantly, a good light, which must remain constant during the shoots, in order to allowing us to make a monotonous video. Avoiding giving any point of attention other that the content itself. This long list was the one that led us to discard this alternative and to choose an Animated Video Editor.

Figure V-1: Doodly Logo[30]

(Reprinted from Doodly Website, of Bryxen (n.d.). Retrieved from https://www.doodly.com/)

Figure V-2: VideoScribe Logo[30]

(Reprinted from VideoScribe Website, of Sparkol Group (n.d.). Retrieved from https://www.vide-oscribe.co/)

For the choice of software, we performed a research among different blogs, YouTube channels and experts' feedback, based on personal opinions and recommendations. Of this investigation two programs stood out: Doodly and VideoScribe.

Both were the most named and the ones with the best overall rankings, so we decided to request the free trial of the two to perform a test on them and determine which alternative best suited our needs. As a result we elaborate a table, in which we compare the advantages and disadvantages of each alternative (see Table V-1).

[30] Screenshot made on 08/09/20

Table V-1: Comparison of Animated Video Editors[31]

Doodly	VideoScribe
Can be purchased online	Can be purchased online
Download is immediate after purchase	Download is immediate after purchase
The free trial is a refund of the purchase until 30 days after the acquisition	Has a free trial of one week
Price standar version: €39 per month	Price standar version: €25 per month
100 Royalty free music tracks	190 Royalty free music tracks
Use on multiple computers	Use only in one computer
High definition (HD) videos in the standar version	No High definition (HD) videos in the standar version
Without Doodly logo watermark on the video	With VideoScribe logo watermark on the video
Allows you to use images outside your catalog and animate them.	Allows you to use images outside your catalog and animate them.
Many images available for user use	Few images available for user use
Easy to animate imported images	Hard to animate imported images
Allows user to import music or voice to video	Allows user to import music or voice to video
Allows record to voiceover	Allows record to voiceover

From all that was observed, the elements that was decisive in the choice was the fact that one had a logo as a watermark in the final video, and the other did not. Added to this was the use of the interface that both products offered to the user which greatly influenced our decision. While VideoScribe's was simple (see Figure V-3), for our preference it was too basic, without leaving the necessary functions at hand, while the Doodly interface (see Figure V-4) seemed more complete and intuitive, with quick access and easier for us to understand and use.

[31] Some information was got from Doodly Website, of Bryxen (n.d.). Retrieved from https://www.doodly.com/ and VideoScribe Website, of Sparkol Group (n.d.). Retrieved from https://www.videoscribe.co/

What especially attracted us to the last option was that it had a bar at the bottom that allowed the user to edit second by second both the audio and the animation, making it similar to other common video editors in which we already had experience from the past.

Figure V-3: VideoScribe Interface

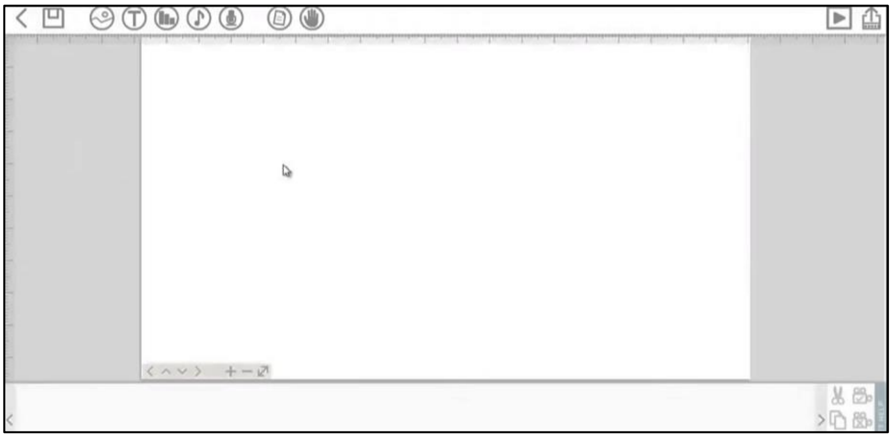
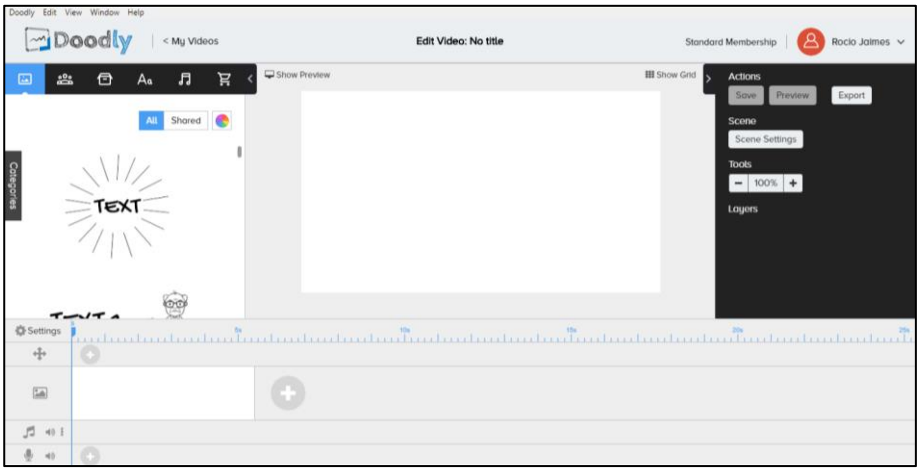


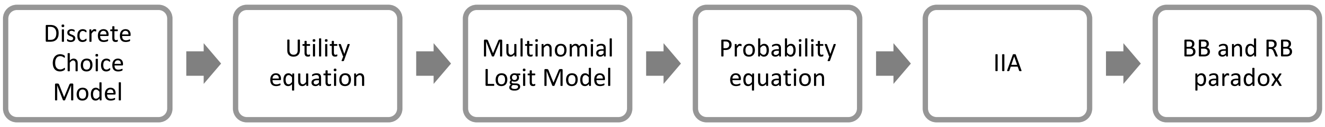
Figure V-4: Doody Interface



5.2. The Script

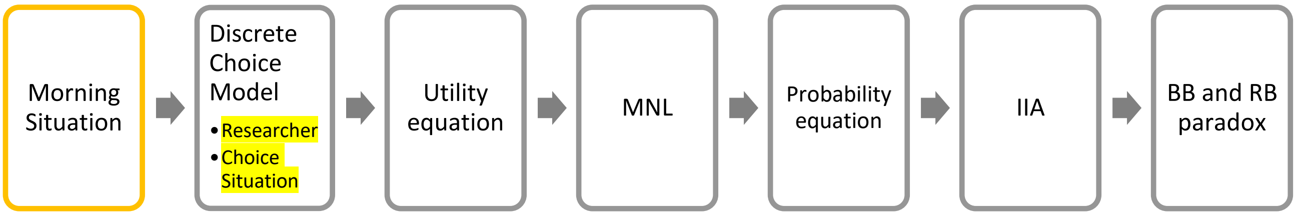
Selected the program to be implemented, what followed was to write a script, the foundation of the entire video. In this we were incredibly careful, trying to avoid leaving important theoretical part out, while the main ideas were organized in sequences. This is how we make a scheme with the line of topics that would be treated, always reviewing the theoretical framework that we developed in the last chapters (for example see Figure V-5).

Figure V-5: Planning line for MNL Video



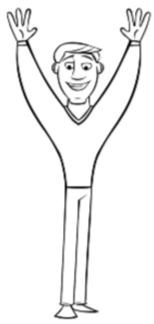
Having the theory ordered the next issue was to think of a way to give dynamism to the video, aiming that the audience could relate the case under study to their everyday situations, and with that accomplish a clearer understanding of the topic. If we could provide a visual imagen mixed with routine scenarios, the chances that the content of our video was long-term remembered were higher.

Figure V-6: Planning line for MNL Video (V.2)



That is why we modified our planned line, adjusting it and beginning with the approach of an initial situation, placed to capture the attention of the spectator (for example see Figure V-6). Next, continuing with our objective of "humanizing" our theory, we proposed an example and created the figure of a researcher, whom we named "Joe" (see Figure V-7), which was used to guide our video, leading the audience from the problematic situation presented, until the paradox occurs due to the IIA.

Figure V-7: "Joe", researcher use as example in the MNL video



(Reprinted from Doodly Software, of Bryxen (n.d.))

With all these ideas, and based on the Planning line, the script was written. After that we proceeded with the reading tests, the audio recordings, and the verification of the length, trying not to exceed six minutes since it has been proven that attention is lost after that time (based on what was developed in the Chapter 2.2). As the reading was done, changes were made to improve the script (see the complete script in the Appendix 1: Video Script section).

Finally satisfied with the result, the next step was to coordinat with the Vocal Actor (VA) where the accentuation would take place and what pauses were necessary to allow the listener to process the information, since they were necessary according to the studies presented in the Chapter 2.2.

5.3. Storyboard and Production

Using the final audio (appropriately recorded and edited) as a guide, a storyboard was designed. This was made in orden to give a clearer idea of what was going to be done later in the edition. For it, vignettes of pictures drawn with pencil were used, which represent the scenes in each one of them. The vignettes were created with two frames in which the scene was drawn at the top, and in the lower part the script was written, to assist in pointing when the scene should change (see Figure V-8 and Figure V-9).

Figure V-8: Storyboard Vignette

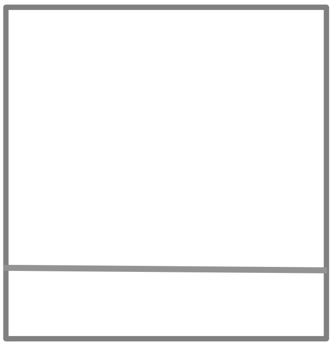
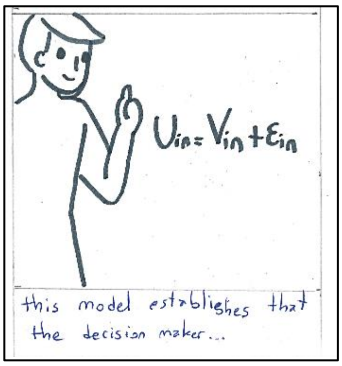


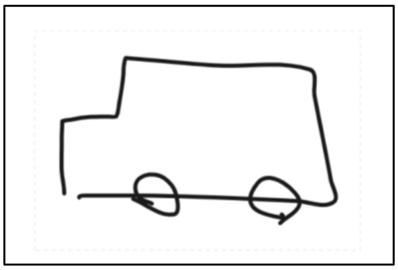
Figure V-9: Storyboard Vignette example



(Part of Appendix 2: Storyboard)

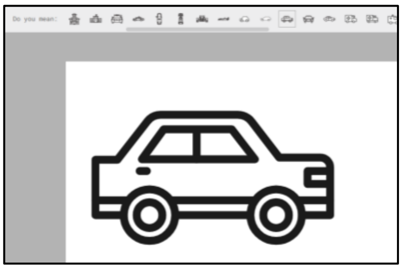
This tool helped speed up the editing process, since all the ideas were organized and planned. It has also contributed to know in advance what elements would be required. As many of these elements were not available in the basic program (especially those in color), they were designed using AutoDraw[32], a website that provides simple outline drawings, ideal for the type of illustration that whiteboard videos require. This website works as follows: the user draws an approximate drawing of what they need (see Figure V-10), the site suggests images that interpret the lines made, and replace the drawing with defined shapes (see Figure V-11) which can be colored and downloaded for free (see Figure V-12).

Figure V-10: AutoDraw example 1[33]



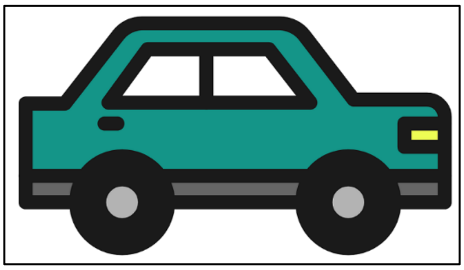
(Screenshot from AutoDraw website, of Google LLC (n.d.). Retrieved from https://www.autodraw.com/)

Figure V-11: AutoDraw example 2[33]



(Screenshot from AutoDraw website, of Google LLC (n.d.). Retrieved from https://www.autodraw.com/)

Figure V-12: AutoDraw example 3[33]



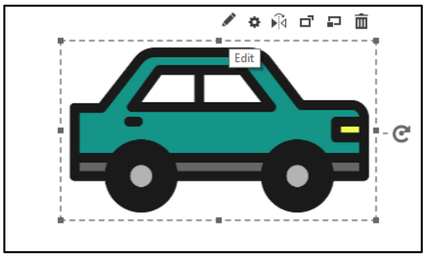
(Screenshot from AutoDraw website, of Google LLC (n.d.). Retrieved from https://www.autodraw.com/)

Said images could be added to Doodly providing us with more elements for the edition. Once incorporated into the video, the drawing hand had to be animated so they would not represent differences with the rest of the animation. Performing manual animations is possible in Doodly, manipulating more specific editing options. The tool allows the user to mark the path of the animated hand, while selecting the way in which the strokes will be executed through the implementation of a series of dots (see

[32] AutoDraw website, of Google LLC (n.d.). Link: https://www.autodraw.com/
[33] Screenshot made on 27/08/20

Figure V-13 and Figure V-14). The result we got was a clear animation, which was concealed with the other images that the program has (see Figure V-15).

Figure V-13: Animation in Doodly example 1

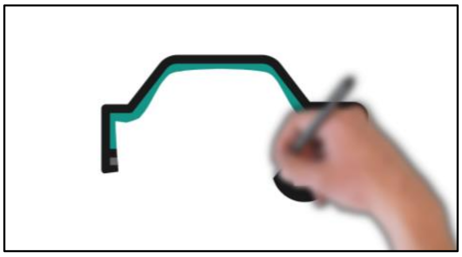


(From Doodly (Bryxen, n.d.))

Figure V-14: Animation in Doodly example 2

(From Doodly (Bryxen, n.d.))

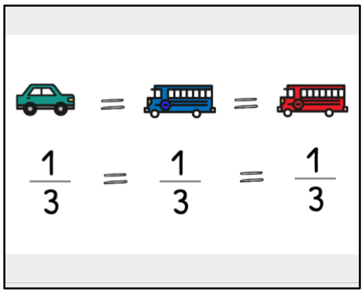
Figure V-15: Animation in Doodly example 3



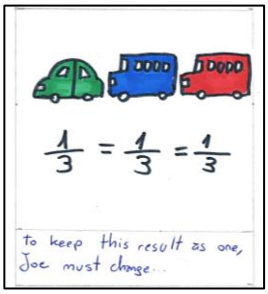
(From Doodly (Bryxen, n.d.))

With so many options available, and tools that allowed adding the missing ones, being able to follow the storyboard almost perfectly was a simple task, but one that took additional time to make it as accurate as possible (see Figure V-16 and Figure V-17 for an example of this). Nevertheless, the fact that the functions and tools were like those of conventional Video Editors (which we had experience, as it was mentioned above) speeded up the time invested. The greatest attention that we had to pay at this stage was taking care of the pauses in the middle of the explanation, leaving enough so that the interlocutor could assimilate the information, but not enough that it could lead to distraction (see Chapter 2.2.2). The coordination between the animation and the recorded voice It was also another point that required special attention.

Figure V-16: Scene in Doodly



(From Doodly (Bryxen, n.d.))



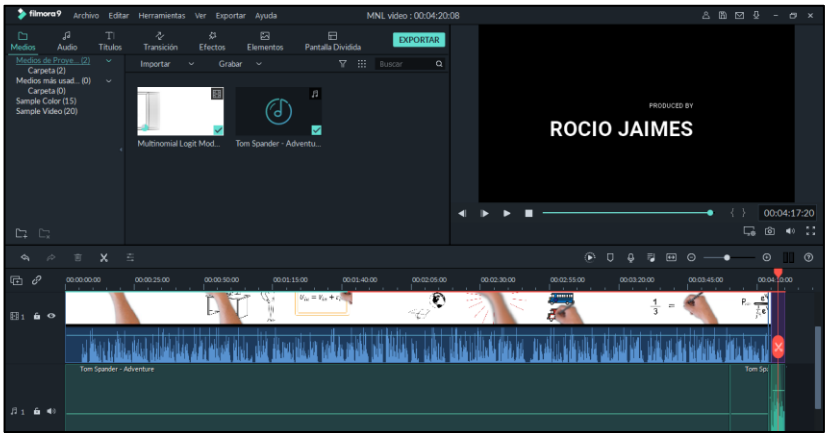
(Part of Appendix 2: Storyboard)

Figure V-17: Scene in Storyboard

5.4. Final Steps and the Finished Product

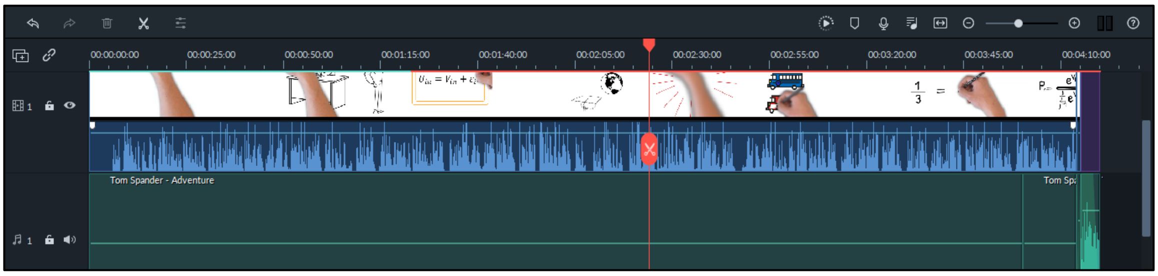
To ensure that all the details of the video were respected, and that the animations and timings were correctly synchronized, numerous renderings of the video were carried out as a test. When the final product met our expectations, Doodly's video was edited one last time with another editing program to add the final details, mainly the background music and the titles (see Figure V-18). For this we chose the editor Wondershare Filmora9 (n.d.).

Figure V-18: Work on Wondershare Filmora9



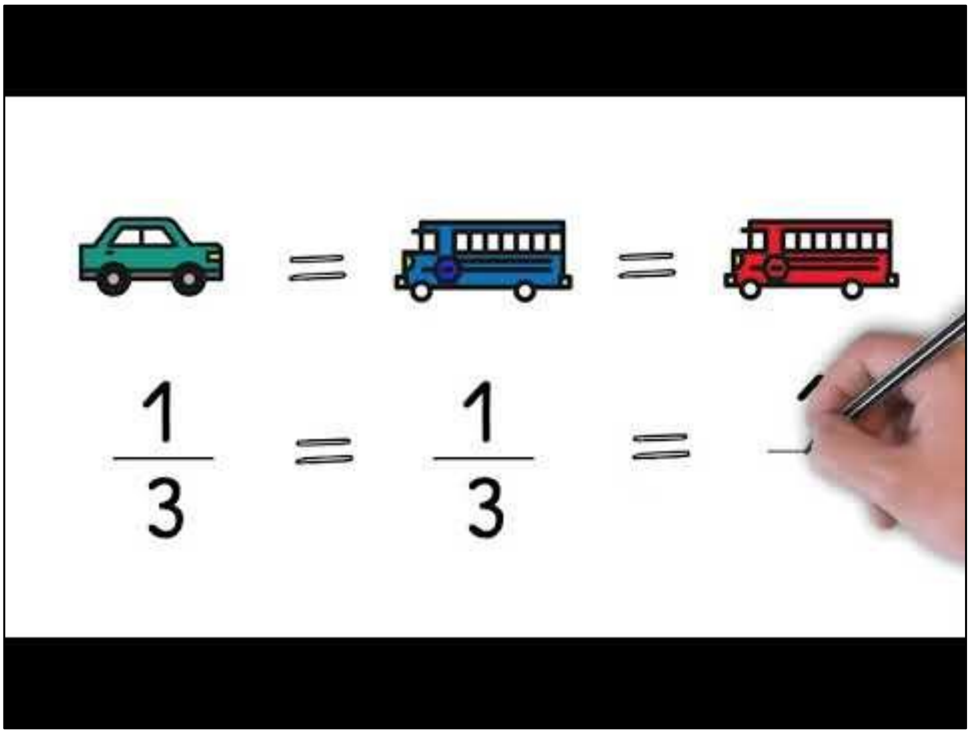
For the background music, quite a few copyright-free audios were downloaded with acoustics according to this type of videos, and a test was carried on then, looking for the one that matches with our project. One of the great advantages of the Filmora editor is that it allows meticulous time management thanks to its time bar (see Figure V-19), which can be zoomed in as close as needed. This helps achieve even more precise sound coordination.

Figure V-19: Wondershare Filmora9 edit bar



When we were adding the music to the video, we had to be incredibly careful that the volume was not loud enough to distract the audience, or that could cover the narrative. The music should, ideally, function as an accompaniment to the VA, covering the small silences that could lose the audience's focus. This editor had better sound management tools than Doodly, so controlling the sound volumes (and even adding variants) has allowed to achieve a professional result (see Figure V-20 for the final product).

Figure V-20: Witheboard Video- MNL: Blue Bus/Red Bus Paradox



(From Appendix 3: Final Video of MNL)

VI. Conclusion

Due to the scenario observed during 2020 and the forced implementation of Virtual Education (VE), we decided to contribute with a video that would be used as a tool by instructors in econometrics classes, aiming to improve the quality of the lessons. Throughout our work we have developed the different types of Educational Videos: Screencast, Talking-Head, Lecture Recording, Simulation and Animated. And from this list an Animated Video was chosen, specifically a Whiteboard production. The choice was made taking into account that the virtual classes were carried out through video conferences or with prerecorded videos whit the Talking-Head or Lecture Recording format. These has proven to be insufficient when when the goal was optimizing education, and basically replaced the TE format.

On the other hand, the Animation Videos have been studied and proved effective due to their dynamism, while the choice of using a Whiteboard was made taking into account the Cognitive Load, in which it established that in order to carry out an adequate transfer from one type of memory to the other, the information must be presented in a non-overloaded way, facilitating its processing and transition. All of this is something that can be achieved with our choice, due to its white background and simple line drawings.

In addition, considerations were made of the other key elements that should be included in the Educational Videos, namely: Cognitive Load, Student Engagement and Active Learning. Implementing it in our experience, the starting point was to capture the attention of the senses and enter the Sensory Memory, use a time of less than six minutes (proven to be optimal) so as not to lose the student's attention, and implement a voice in the narration capable of transmitting emotions and empathy we concluded that our video should be able to pass to the Working Memory. But to achieve the transition to Long-Term Memory other tools are necessary and must be carried out by the student himself.

As it was also developed, the use of a single type of education (remember TE and VE) was always proven inefficient compared to implementing a mix, called HE. Students achieve better grades and better understanding if the TE is used and complemented with videos, which reinforce definitions or detail key points. We emphasize this since, as shown in studies cited, the use of only a video (like the one we developed) proves to be inefficient without the proper complement of a class.

Then, in our thesis we continue developing a theoretical framework more specific to the topic that was explained in our production: DCM concepts, its elements, the Utility Function and the ε term. Continuing with MNL, its properties, Probability Equation, the IIA and the Blue Bus/Red Bus paradox. All this was analyzed to carry out the script, and later the storyboard of the video. The focus was made on the IIA as it is a distinctive property of the MNL, and the paradox, since is a case used in many academic texts.

6.1. Achieved results and future development

After analyzing which edition program would be implemented, Doodly was chosen. Then the script was written and voice recording tests were carried out, where the pauses and the tone of voice applied were controlled. Finally, a storyboard was drawn, which served as a guide throughout the edition. The selected tool (Doodly) allowed the incorporation of external elements, which made it easier to express the idea in a similar way as planned. Finally, the Filmora9 program was used to carry out the final audio/animation synchronization.

The result was a Whiteboard Video, that meets the elements of all optimal educational video: With a charismatic voice narrator which the interlocutor can feel in affinity, without many pauses that lead to boredom and distraction, with a length of less than six minutes (four minutes and twenty seconds, to be more precise), with mention of everyday situations that make it easy to remember the content, and a clear animation according to the content.

Theoretically speaking our work meets the optimal requirements, but its effective result remains to be tested in the future. However, we consider that we have provided a useful tool for education, which can optimize Traditional or even Virtual classes. We believe that the combination of education with technology is the future (or the present, demonstrated by the context) since it facilitates learning and dynamics. We hope it begins to be applied as a new standard.

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Appendix

Appendix 1: Video Script

Let us take an everyday situation, we get up in the morning and decide how we will go from home to work.

Maybe by bicycle? Will we use the train? the bus? The decision we make will be relevant to various government agencies such as the transportation planning department. Knowing the flow of users can help to program the public service according to the needs of the population.

Suppose that a researcher, whom we will call "Joe", wants to find out how many people use the different means of transportation to commute to work. The population can only choose between going by car or taking a blue bus.

Joe chooses a discrete choice model since the alternatives presented are listed in a finite list. This model establishes that the decision maker will always choose the best option, which is why it is formed by a utility maximization equation:

$$U_{in} = V_{in} + \varepsilon_{in}$$

Where the true utility of the alternative *i* to the decision maker *n* equals *V* plus epsilon.

The equation can be separated into two parts, one deterministic and the other stochastic. Vit will be formed by the characteristics of the decision maker (for example, their income), the attributes of the alternatives (for example, travel time) and how the individual evaluates each alternative: for a person with low income the cost will be a determining factor, while for a person with high income the important attribute will be the time.

On the other hand, ε will be formed by all those elements that the researcher cannot analyze or does not consider but had an influence in the decision. Since Joe is applying a Multinomial Logit Model, ε is distributed independently and identically among the alternatives.

Now let us move on to the formula, which will be represented by:

$$P_{in} = \frac{e^{V_{in}}}{\sum_{j=1}^{J} e^{V_{jn}}}$$

where the probability of an alternative equals the exponential of the utility of said alternative, divided by the sum of the exponentials of the total choice set. In our example this would be something like:

$$P_C = \frac{e^{V_C}}{e^{V_C + e^{V_{BB}}}} \text{ and } P_{BB} = \frac{e^{V_{BB}}}{e^{V_C + e^{V_{BB}}}}$$

Numerically, and for simplicity, we suppose that the probability of choosing any, the car or the bus, is one-half:

$$P_C = \frac{1}{2} \text{ and } P_{BB} = \frac{1}{2}$$

The MNL has a distinguished property, it is called the independence of the irrelevant alternatives (IIA), which establishes that if we compare two alternatives, the probability of choosing between these two is going to be independent of the attribute of others. In other words, the probability of the comparison will remain constant until one of the elements is modified. For our example it would look something like this:

$$\frac{P_C}{P_{BB}} = \frac{1/2}{1/2} = 1$$

Now suppose that a new bus joins the choice set, this time in red, which will have exactly the same attributes as the blue bus: same route, stops, cost and schedule, the only difference will be the color. What would then be the probability for our 3 alternatives?

The logical thing to do for Joe would be to split the probability corresponding to the blue bus, while the probability of the car remains unchanged.

$$\frac{1/2}{2} = \frac{1}{4} = P_{BB}, P_{RB}$$

But this would violate the property of IIA, since now the probability of the blue bus has changed and the comparison with the probability of the car is no longer one.

$$\frac{P_C}{P_{BB}} = \frac{1/2}{1/4} = 2 \neq 1$$

To keep this result as one, Joe must change the probabilities of all the alternatives.

$$P_{BB} = P_{RB} = P_C = \frac{1}{3}$$

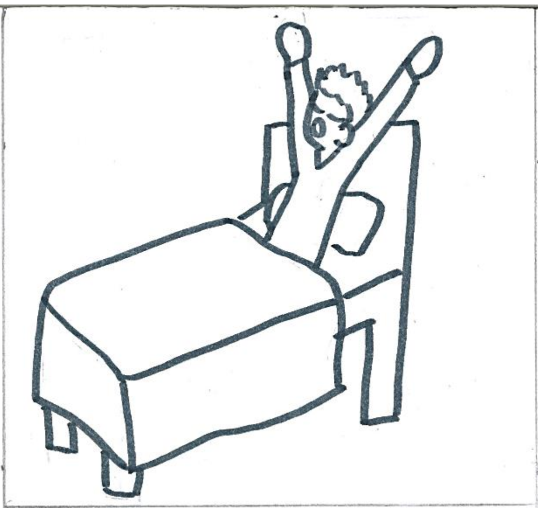
In this way the IIa is respected, but it no longer represents reality.

Of course, this is an extreme case, elaborated to show that the MNL can overestimate the probability, leading to an error. This happens when a change occurs in the choice set, either because a new option is available, one is no longer so, or maybe a change in the Attribute of the Alternative happens, such as the price rising.

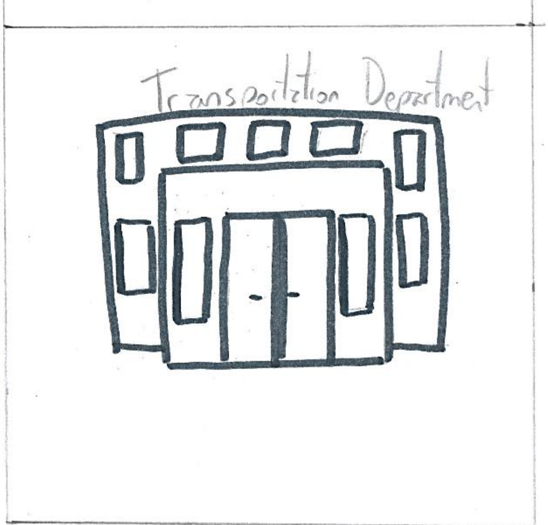
Despite the paradox that can be caused, the model itself is complete, easy to use and interpret the results. The error should not be significant, as long as the information is correct, and the population is properly segmented.

That is all, thank you for watching.

Appendix 2: Storyboard

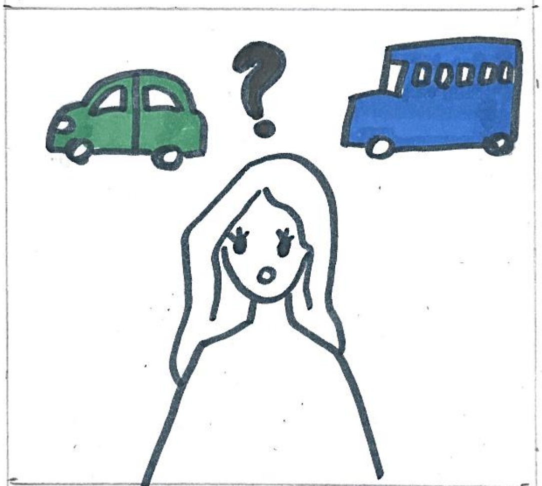


Let us take an everyday situation...

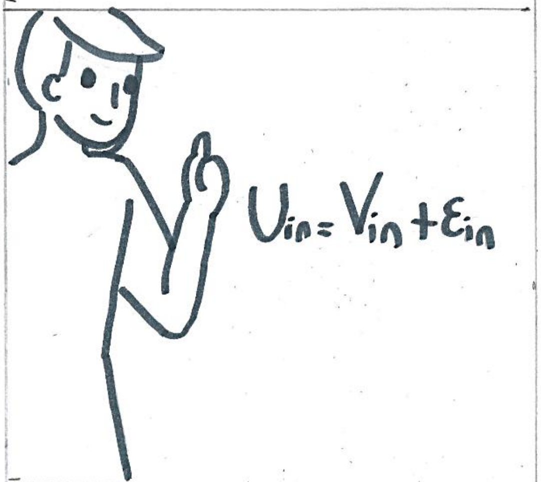


the decision we make will be relevant...

Suppose that a researcher, Joe ...

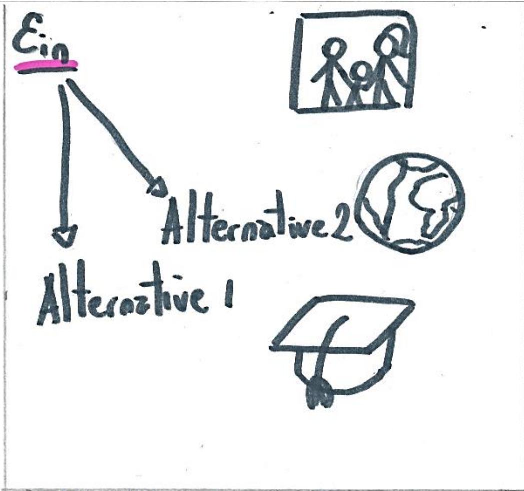


Joe choose a discrete choice model...

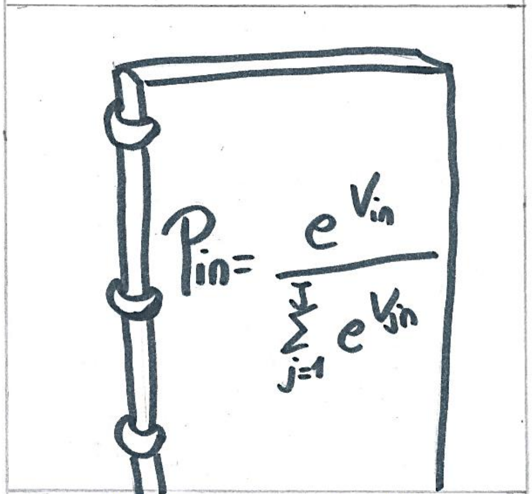


this model establishes that the decision maker...

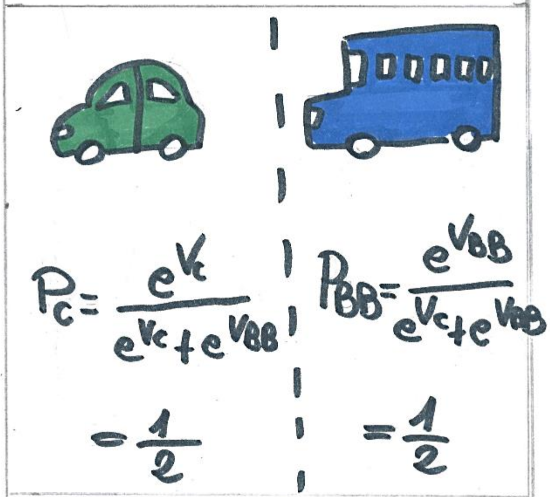
V_{in} will be formed by the characteristics...



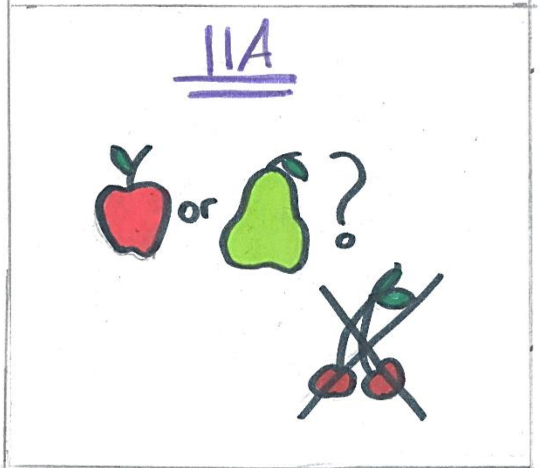
On the other hand, ε_{in}



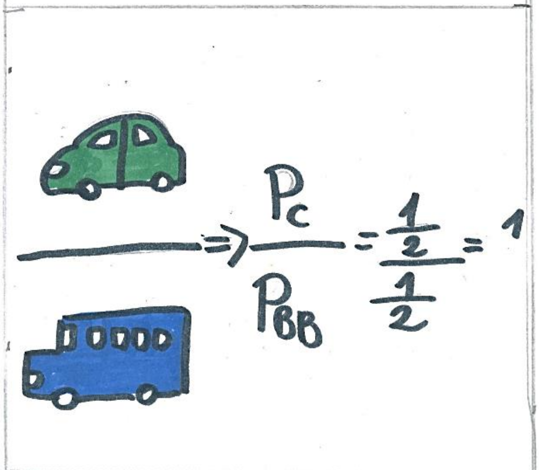
Now, let us move on to the formula, which will be...



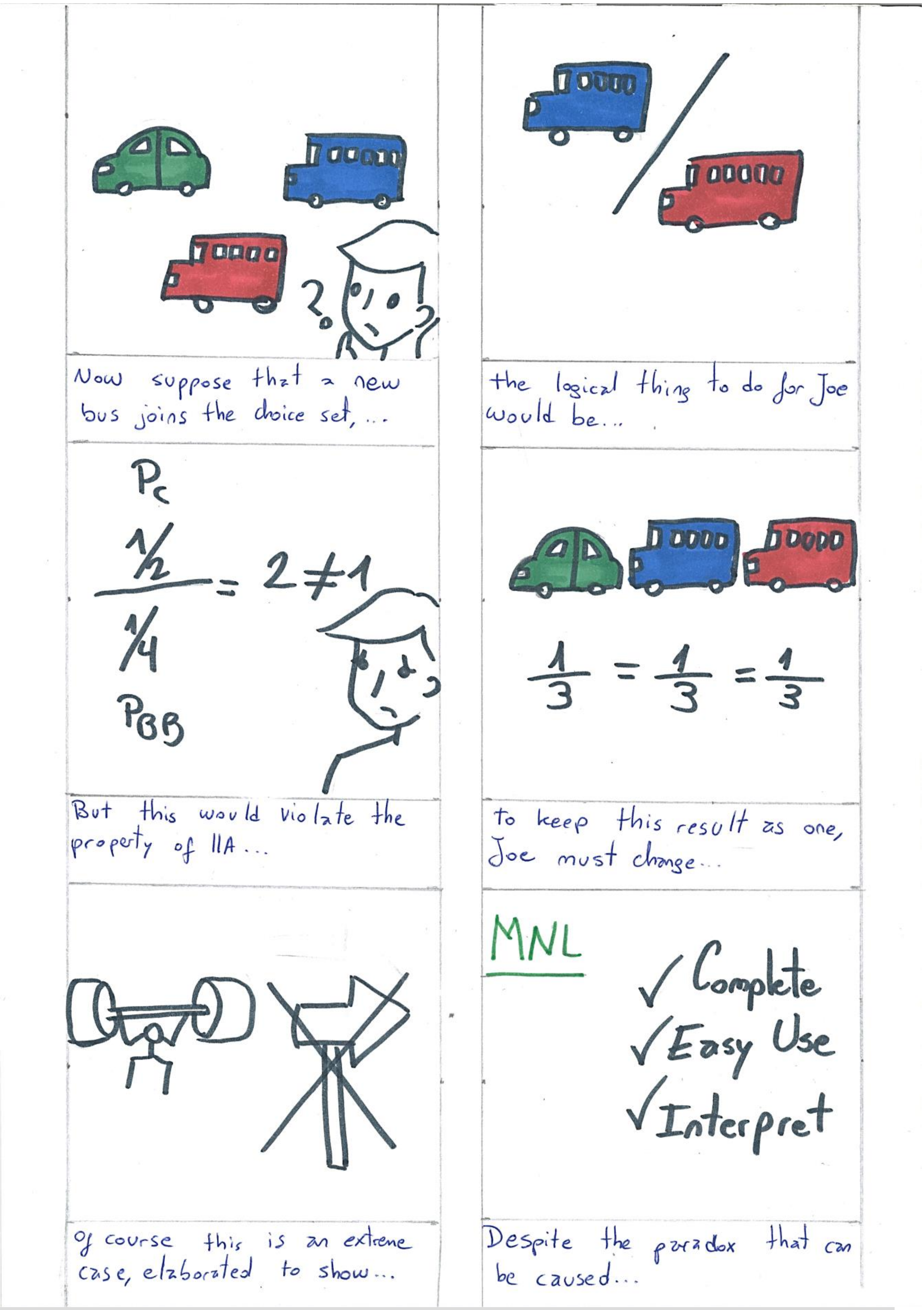
In our example this would be something...

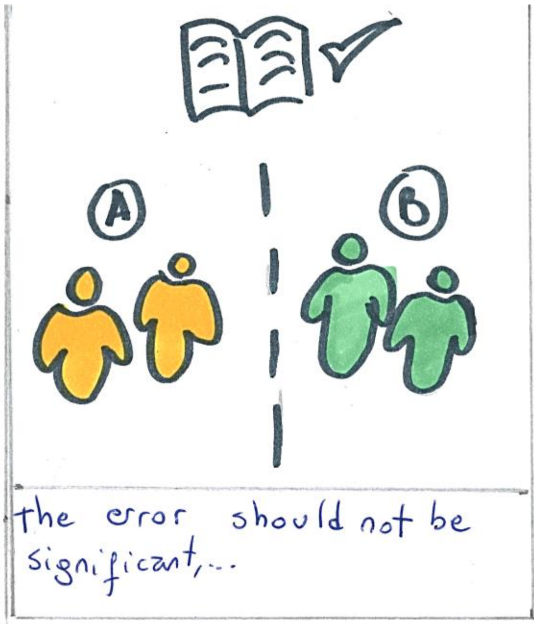


the MNL has a distinguished property, IIA

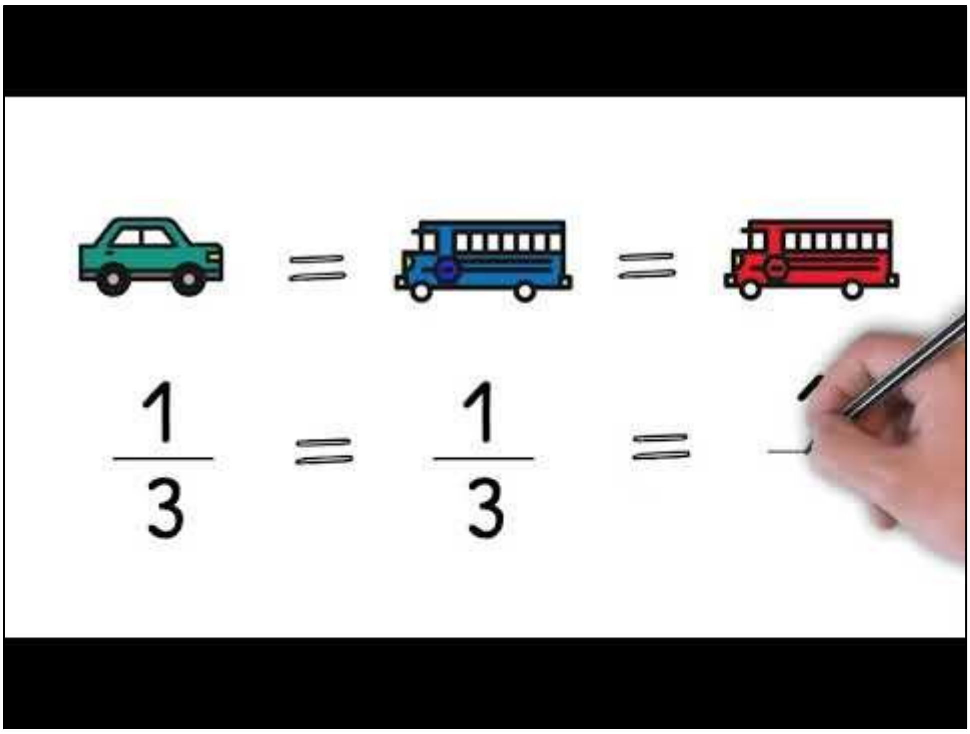


For our example it would look something like...


?
Now suppose that a new bus joins the choice set, ...
the logical thing to do for Joe would be...
Pc
1/2
= 2 ≠ 1
1/4
PBB
But this would violate the property of IIA...
1/3 = 1/3 = 1/3
to keep this result as one, Joe must change...
Of course this is an extreme case, elaborated to show...
MNL
✓ Complete
✓ Easy Use
✓ Interpret
Despite the paradox that can be caused...



Appendix 3: Final Video of MNL



Statement

I hereby affirm that I have completed the present *Master thesis* with the topic:

“EXPLAIN DISCRETE CHOICE METHODS BY ANIMATION VIDEOS”

And it was done by myself without the use of any means or sources other than those indicated. The passages which have been taken from other works, either in their wording or in their sense, have been marked as borrowings in each individual case by stating the source, including the secondary literature used. And I am aware: In case of usage of internet sources I should indicate such sources and add a dated print out of the web page including the URL to the Reference list of the *Master Thesis.*

This thesis was not submitted to any other examination committees.

Place, Date

Frankfurt (Oder), Germany/ 25.09.2020

Signature